A NEW METHOD TO ANALYZE TRANSIENT STATE SIGNAL OF POWER SYSTEM

A Project

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Yijiao Liu

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Date

Department of Electrical and Electronic Engineering
Abstract

of

A NEW METHOD TO ANALYZE TRANSIENT STATE SIGNAL OF POWER SYSTEM

by

Yijiao Liu

Statement of Problem

Power systems are transitioning into a new era by focusing more on the system characteristics like transient signals. Due to the demand of a more effective way to extract the fault line transient signals, many new methods are introduced into power system to solve the signal analysis problem in order to catch the main characteristics of status changes. Hilbert transform method as the foundation of solving such issues is widely used nowadays. As a result, it is essential to come up with a transfer method that can help analyze even the small changes of the system and can tell the physical significance of such changes.

This project is a continuation of a previous work based on a research of transient signal analysis for proposing a method to extract signal changes that can be used in fault line
detection and find the fault location. It will focus on the main characteristics of Hilbert transform method and then come up with a new method, the generalized sinusoidal transfer method that can help describe the signals' small changes.

**Sources of Data**

The simulations are done using Matlab and Simulink toolbox, and the models of fault line system are from several published research papers.

**Conclusions Reached**

To analyze power system fault signals using the generalized sinusoidal method can help improve the accuracy and efficiency of system analysis. And the new proposed method is of great value to extract transient power system signal and find the fault line and fault location.

_________________________, Committee Chair

Jian Luo, Ph.D.

12/04/15

Date
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1.1. Background of the Project

Signal is the carrier of information and energy. Signal analysis is a research of the essential attribute for signals. Generally speaking, signal is a function that has relationship with time and space. Due to the variety forms of signal, it can be represented in many ways. Among them, a unique form that can help people has better understanding of the characteristics and extract what they need seems to be very important\cite{1}\cite{2}. As a mixture of electrical engineering, system engineering and applied mathematics, there are many accesses to deal with signal processing.

In the theoretical research and engineering practice of the power system, signal detection and extraction, analysis, and judgment are of great significance. Whether operation and control under steady state conditions, or protection and safety monitoring in the electromagnetic transient, we need to extract information that reflects the dynamic characteristics of the system quickly and accurately. Most of traditional methods mainly focus on extracting the electric parameters in power frequency; the information extraction method and features' diversity are deficiencies obviously. Though power system transient state contains an exuberant amount of information, there is no effective mathematical method to help solve these transient problems makes it difficult to do further researches. Therefore, to develop a new signal processing method to analyze the signals is badly needed for theoretical research and engineering practice.
As is known, non-ground neutral system and arc-suppression-coil-ground neutral system are called non-direct ground neutral systems. These two system models are widely used in power distribution systems. When it comes to the types of distribution system faults, the single-phase earth fault of the undercurrent earth system has the highest breakdown probability, and approximately composes 80% of all breakdowns numbers. Specifically, this failure may causes sudden increase of the line current and electric arc is hard to extinguish. If the system is about to run a long time under this condition, the fault will extend to a cascade shutdown accident. Therefore detecting the fault line correctly is important to clear the fault and improve power supply reliability when single-phase fault occurs in neutral indirectly grounded systems.

In this project, a new method of fault detection is presented based on deep study of phase-to-ground fault transient conditions and the generalized sinusoidal transform method.

1.2. Advantages of Generalized Sinusoidal Transform Method

This new method adopts a new standpoint, the amplitude and phase are extended from steady state to transient field. Power system signals are observed to obtain the instantaneous characteristics; the instantaneous amplitude and instantaneous phase can help describe power system features. This method is established on strict mathematical basis, explaining the important properties and detailed proof and presenting a calculation method, so that the practical application can go smoothly.
1.3. Goals of the Project

With the increasing demand of a more accurate data extraction, more research and developed projects are needed for finding new ways to analyze the dynamical power system in the future grid. As a continuation of the previous work that presented a method based on traditional sinusoidal transform method\(^2\), this project intends to extend the previous work by construct a pair of primary functions \(\Phi(t)\) and \(\Psi(t)\), to transform the primary function to generalized sinusoidal function with instantaneous amplitude and phase. By discussing transient zero current and the method of analyzing it, a single phase to ground model is built to test the correctness and usefulness of the generalized sinusoidal transform method.

1.4. Layout of the Project

In this project, a method of the transient signal analysis is proposed based on Hilbert transform method. After the Hilbert transform method is discussed in detail, generalized sinusoidal transform method is talked about in chapter 2, including the specific transform steps and its unique characteristics that are of great use for reality applications. In the following sections, several common signals in power system are listed and then the new method of generalized sinusoidal transform method is used to help analyze each kind of the signal. On the one hand I want to test the correctness of this new proposed method, on the other hand I want to use the consequence of the comparison between the test and the theory to help find a useful new way of signal analysis.

The project is organized as follows: The development of generalized sinusoidal transform method and its unique characteristics are discussed in chapter 2. Detailed use of the new
method to help analyze different kind of signals in power systems is presented in chapter 3. Simulation results are all presented in this chapter. Then, in chapter 4, the promising application is talked about and single-phase to ground fault of arc-suppression-coil-ground neutral system is discussed in detail. The conclusions and future work are presented in chapter 5.
CHAPTER 2
GENERALIZED SINUSOIDAL TRANSFORM METHOD

There are many methods to help deal with classic signals, including time domain analysis, frequency domain analysis and time-frequency analysis. For time domain analysis, we are familiar with least square method, Kalman filter method and Prony method, and for frequency domain analysis, Fourier analysis is the most important one. With the growing demand of accuracy in power system signal analysis, time-frequency analysis has become a popular topic in recent years, especially the Wavelet transform method.

In this chapter, a new method is proposed to help analyze signals in power systems based on Hilbert transform method, and then comparisons are made with other method that exist currently.

2.1. Hilbert Transform

Hilbert transform is one of the most important theoretical methods for signal analysis and processing. Hilbert transform method enjoys many special properties, for example, after transforming, a time domain (or frequency domain) signal will still remain in the same domain\(^3\). For any function \(f(t)\) in the time domain, Hilbert transform can be written as \(h(t) = \mathcal{H}[f(t)]\). Given below in equation (1) are the equations relating a time function and its Hilbert Transforms:
Then in the frequency domain, the response of the spectrum function is

\[ H(\omega) = -j \text{sgn}(\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases} \] (2)

When \( H(\omega) = |H(\omega)| e^{j\varphi(\omega)} \), we get

\[ \begin{cases} |H(j\omega)| = 1 \\ \varphi(\omega) = \begin{cases} -\frac{\pi}{2}, & \omega > 0 \\ \frac{\pi}{2}, & \omega < 0 \end{cases} \end{cases} \] (3)

The system diagram is shown below in Figure 1:

Now we can see that the physical interpretation of Hilbert transform is a time-delay filter. The magnitude characteristic of the signal \( f(t) \) remains the same, while the phase character changes. According to that, the component of positive frequencies changes its phase of \(-90^\circ\), the component of negative frequencies changes its phase of \(+90^\circ\).

Several important characteristics of the Hilbert transform are listed below:

1. Linearity

Suppose that \( a \) and \( b \) are random numbers, and \( h_1(t) = \mathcal{H}[f_1(t)], \ h_2(t) = \mathcal{H}[f_2(t)] \), Then we can get

\[ h(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{f(\tau)}{t - \tau} d\tau \]

\[ f(t) = \frac{-1}{\pi} \int_{-\infty}^{+\infty} h(t) d\tau \]
\[ ah_1(t) + bh_2(t) = H[af_1(t) + bf_2(t)] \] 

(4) Shifter

The magnitude character of the signal \( f(t) \) remains the same, while the phase character changes.

(3) Conservation of Energy

According to the Parseval theory, the original signal and its frequent function have the same amount of energy

\[ E = \int_{\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega \] 

(5)

(4) Analytic Signal

Hilbert transform is usually used to build analytic signals. If we define analytic function as \( z(t) = f(t) + jh(t) \), then use Hilbert transform, we get

\[ Z(j\omega) = \begin{cases} 2F(j\omega), & \omega > 0 \\ 0, & \omega < 0 \end{cases} \] 

(6)

2.2. Theoretical Foundation

The aim of generalized sinusoidal transform method is to construct a pair of primary functions \( \Phi(t) \) and \( \Psi(t) \), and then transform the primary function to generalized sinusoidal functions with instantaneous amplitude and phase. To put instantaneous amplitude and phase into mathematical expression, we proceed as follows

\[ f^*(t) = A_m(t) \sin[\omega_0 t + \varphi(t)] \] 

(7)

Then we can compare equation (7) with classical standard sinusoidal function
\[ f(t) = A_m \sin[\omega t + \varphi] \quad (8) \]

From the above, we can see that, in the classical standard sinusoidal function, three elements, amplitude, phase and frequency are constant. Through these elements, we can determine a standard sinusoidal function. At the same time, in the generalized sinusoidal transform function, only frequency remains unchanged, both the amplitude and phase change with time. In this form, we define \( A_m(t) \) as instantaneous amplitude and \( \varphi(t) \) as instantaneous phase.

2.3. Implementation Method

For any original signal \( f(t) \), we can get a new form \( h(t) \) after Hilbert transform

\[ h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau} \, d\tau \quad (9) \]

Then we can construct a pair of primary functions \( \Phi(t) \) and \( \Psi(t) \) with the help of standard sinusoidal function \( \sin \omega t \) and \( \cos \omega t \) as follows:

\[
\begin{align*}
\Phi(t) &= f(t) \cos \omega t + h(t) \sin \omega t \\
\Psi(t) &= f(t) \sin \omega t - h(t) \cos \omega t
\end{align*}
\]

(10)

In the same way, the original signal \( f(t) \) and its Hilbert transform \( h(t) \) can be written as

\[
\begin{align*}
f(t) &= \Phi(t) \cos \omega t + \Psi(t) \sin \omega t \\
h(t) &= \Phi(t) \sin \omega t - \Psi(t) \cos \omega t
\end{align*}
\]

(11)

2.3.1. Analysis of Generalized Sinusoidal Transform Method

First we can imagine a standard sinusoidal function, \( f(t) = \sin \omega_0 t \). With the help of triangle transform, we can change \( f(t) \) into another form with definite frequency as follows:
\[ f(t) = (\sin \omega t \cos \omega_0 t - \cos \omega t \sin \omega_0 t) \cos \omega_0 t + (\cos \omega t \cos \omega_0 t + \sin \omega t \sin \omega_0 t) \sin \omega_0 t \]  

(12a)

According to the Hilbert transform characters, we know that \( f(t) = \sin \omega_0 t \) can be changed to \( f^*(t) = -\cos \omega_0 t \). Then the above equation can be written as

\[ f(t) = \left( f(t) \cos \omega_0 t + f^*(t) \sin \omega_0 t \right) \cos \omega_0 t + \left( f(t) \sin \omega_0 t - f^*(t) \cos \omega_0 t \right) \sin \omega_0 t \]  

(12b)

If we set

\[
\begin{align*}
A(t) &= f(t) \cos \omega_0 t + f^*(t) \sin \omega_0 t \\
B(t) &= f^*(t) \sin \omega_0 t - f(t) \cos \omega_0 t
\end{align*}
\]

(13)

The original equation will be changed to

\[ f(t) = A(t) \cos \omega_0 t + B(t) \sin \omega_0 t \]  

(14)

From this equation we can see that after the transform of original signal, the form of the signal changes to a standard sine function multiplied by another form of function. Usually, if the original signal is given in a definite form, we can no longer use triangle transform and change the equation in the above way, but we can still use Hilbert transform and construct the primary functions.

\[
\begin{align*}
\Phi(t) &= f(t) \cos \omega t + h(t) \sin \omega t \\
\Psi(t) &= f(t) \sin \omega t - h(t) \cos \omega t
\end{align*}
\]

(15)

Putting equation (15) into matrix form:

\[
\begin{bmatrix}
\Phi(t) \\
\Psi(t)
\end{bmatrix} = \begin{bmatrix}
\cos \omega t & \sin \omega t \\
\sin \omega t & -\cos \omega t
\end{bmatrix} \begin{bmatrix}
f(t) \\
h(t)
\end{bmatrix}
\]

(16)
Let \( M = \begin{bmatrix} \cos \omega t & \sin \omega t \\ \sin \omega t & -\cos \omega t \end{bmatrix} \), according to algorithms of advanced algebra, the inverse matrix of \( M \) is given by:

\[
M^{-1} = \frac{M^*}{|M|} = \begin{bmatrix} \cos \omega t & \sin \omega t \\ \sin \omega t & -\cos \omega t \end{bmatrix}
\]

(17)

So the equation can be written as

\[
\begin{bmatrix} f(t) \\ h(t) \end{bmatrix} = \begin{bmatrix} \cos \omega t & \sin \omega t \\ \sin \omega t & -\cos \omega t \end{bmatrix} \begin{bmatrix} \Phi(t) \\ \Psi(t) \end{bmatrix}
\]

(18)

or, extracting from the matrix:

\[
f(t) = \Phi(t) \cos \omega t + \Psi(t) \sin \omega t
\]

(19)

\[
f(t) = \sqrt{\Phi^2(t) + \Psi^2(t)} \left\{ \frac{\Phi(t)}{\sqrt{\Phi^2(t) + \Psi^2(t)}} \cos \omega t + \frac{\Psi(t)}{\sqrt{\Phi^2(t) + \Psi^2(t)}} \sin \omega t \right\}
\]

(20)

\[
\left\{ \begin{array}{l}
A_m(t) = \sqrt{\Phi^2(t) + \Psi^2(t)} \\
\varphi(t) = \arg \frac{\Phi(t)}{\Psi(t)}
\end{array} \right.
\]

(21)

Finally \( f(t) = A_m(t) \sin [\omega t + \varphi(t)] \).

The flow diagram of generalized sinusoidal transform method is given below in Figure 2:
1. Do Hilbert transform to f(t) to get h(t)
2. Get primary functions from f(t) and h(t)
3. Use primary functions to represent original function
4. Deduce the expression of generalized sinusoidal function
5. Give definition to transient magnitude and transient phase
6. Analyze the characteristics of signal and put it into application

Figure 2. Flow diagram of generalized sinusoidal transform method

The diagram above is the basic way of executing this generalized sinusoidal transform method. To make a conclusion of this method is to do the Hilbert transform to get the primary functions; then build the original signal based on the primary functions. Finally we get the generalized sinusoidal transform from mathematical manipulation.

2.3.2. Characteristics of Generalized Sinusoidal Transform Method

1. Primary function \( \Phi(t) \) and \( \Psi(t) \) are Hilbert transform to each other.

\[
\begin{align*}
\Psi(t) &= \mathcal{H} \left[ \Phi(t) \right] \\
\Phi(t) &= \mathcal{H}^{-1} \left[ \Psi(t) \right]
\end{align*}
\]  

2. Primary functions \( \Phi(t) \) and \( \Psi(t) \) have equal energy and are mutually orthogonal
\[
\begin{align*}
\int_{-\infty}^{\infty} \left[ \Phi(t) \right]^2 - \left[ \Psi(t) \right]^2 dt &= 0 \\
\int_{-\infty}^{\infty} \Phi(t) \cdot \Psi(t) dt &= 0
\end{align*}
\] (23)

3. Linearity

If \( f_1(t) = \Phi_1(t) \cos \omega t + \Psi_1(t) \sin \omega t \), \( f_2(t) = \Phi_2(t) \cos \omega t + \Psi_2(t) \sin \omega t \), then

\[
f_1(t) + f_2(t) = \left[ \Phi_1(t) + \Phi_2(t) \right] \cos \omega t + \left[ \Psi_1(t) + \Psi_2(t) \right] \sin \omega t
\]

4. Transformation matrix \( M = \begin{bmatrix} \cos \omega t & \sin \omega t \\ \sin \omega t & -\cos \omega t \end{bmatrix} \) is an orthogonal matrix.

\[
M^T M = \begin{bmatrix} \cos \omega t & \sin \omega t \\ \sin \omega t & -\cos \omega t \end{bmatrix} \begin{bmatrix} \cos \omega t & \sin \omega t \\ \sin \omega t & -\cos \omega t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = E
\] (24)

2.4. Comparison between Different Methods

The section above gives detailed analysis of generalized sinusoidal transform method. In order to show the applicability and effectiveness of this new method, we list many methods that are commonly used to analyze signals to make comparison of these methods.

2.4.1. Generalized sinusoidal transform method

This method first tries to construct two primary functions on the basis of Hilbert transform method of the original signal \( f(t) \). Then getting the expressions of transient magnitude \( A_m(t) \) and transient phase \( \varphi(t) \), we can catch even small changes in magnitude by using the concept of transient magnitude \( A_m(t) \). At the same time, transient phase can also help to describe original signal's initial phase. In this way, this generalized
sinusoidal transform method can extract the magnitude and phase at any time, and the meaning of transient magnitude and transient phase is also very clear.

2.4.2. Least square method[4]

People like to use least squares method to help analyze signals in the time domain; however, we have to admit that this least squares method has many difficulties in practical applications. First we have to set a function model, and the fitting function must be added to a signal that has the same expression of the original signal. From the previous research we can see that this method can match the tendency of the original function, but it has the disadvantage on precision, that is to say the least square method must be used in the whole time domain. It cannot reflect the sudden changes of the function due to the constant parameters.

2.4.3. Fourier transform[5][6]

The classic technique of analyzing signals in the frequency domain is the Fourier transform. After taking Fourier transform of the original signal \( f(t) \), we get the corresponding frequency function \( F(n) \). This method faces the same problem with the least squares method, in that it cannot reflect the sudden changes of signal. The integral action of the Fourier transform smoothens the sudden changes of the function. And this method does transforms for the whole signal, the result of it is to get the average value of magnitude and phase, which, sometimes may lead to estimation error.

2.4.4. Wavelet transform method[7][8]
Wavelet transform method can yield the signal changes during the specific period of time; however, it cannot tell how this change happens or what is behind the change. Hence it has disadvantage of not sufficiently yielding detail of changes or transient changes. From this we can see that analysis of the signal using the time-frequency method cannot extract characteristics of transient magnitude and transient phase.
CHAPTER 3
POWER SYSTEM SIGNAL ANALYSIS BASED ON GENERALIZED SINUSOIDAL TRANSFORM METHOD

In this chapter, the generalized sinusoidal transform method will be used to help analyze power system signals. First, we can divide signals into three categories, single-component signal, combined signal and complex signal, as shown below in Table 1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Specific signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-component signal</td>
<td>Power-frequency steady-state signal</td>
</tr>
<tr>
<td></td>
<td>High-frequency steady-state signal</td>
</tr>
<tr>
<td></td>
<td>Low-frequency steady-state signal</td>
</tr>
<tr>
<td></td>
<td>Decaying DC signal</td>
</tr>
<tr>
<td></td>
<td>Power-frequency decaying DC signal</td>
</tr>
<tr>
<td>Combined signal</td>
<td>Choose any two of the single-component signal</td>
</tr>
<tr>
<td>Complex signal</td>
<td>More complex than combined signals</td>
</tr>
</tbody>
</table>

Table 1. Typical signals of power system

3.1. Single-component Signal

In this part, several signals will be considered as examples to illustrate the correctness of the generalized sinusoidal transform method.

3.1.1. Power-frequency steady-state signal

First we set the original signal as \( f(t) = \sin(\omega t + \frac{\pi}{3}) \), \( \omega = 2\pi \times 50 \), \( f_s = 10kHz \), with the sampling window time as 100ms. We know that the signal will take the form of \( f^*(t) = A_m(t)\sin[\omega^* t + \varphi(t)] \), in which the \( \omega^* = 2\pi \times 50 \).
Figure 3. Diagram of power-frequency steady state signal

From Figure 3, we can see that the frequency of the signal $f(t)$ and $h(t)$ does not change after the transform; Hence transient magnitude $A_m(t)$ and transient phase $\varphi(t)$ are all constant. Under this condition, generalized sinusoidal transform method transforms the original signal into standard sinusoidal signal. The simulation results are in correspondence with theoretical results.

3.1.2. High-frequency steady-state signal

First we set the original signal as $f(t) = \sin(200\pi t + \frac{\pi}{6})$, $\omega = 2\pi \times 50$, $f_s = 10kHz$, and the sampling window time is 100ms. We know that the signal will turn into the form of $f^*(t) = A_m(t) \sin \left[ \omega' t + \varphi(t) \right]$, in which the $\omega' = 2\pi \times 50$. 
Figure 4. Diagram of high frequency steady state signal

From Figure 4, we can see that generalized sinusoidal transform method can extract high-frequency steady state signal's primitive characters. At the time when \( t=0 \), the initial phase of signal \( f(t) \) is \( \phi = \frac{\pi}{6} \), and in the monotonically increasing section \( \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \), the frequency of original signal \( f(t) \) is higher than the frequency of \( f^*(t) \).

3.1.3. Low-frequency steady-state signal

First we set the original signal as \( f(t) = \sin(40\pi t + \frac{\pi}{6}) \), \( \omega = 2\pi \times 50 \), \( f_s = 10kHz \), and the sampling window time is 100ms. We know that the signal will turn into the form of \( f^*(t) = A_m(t)\sin[\omega^*t + \phi(t)] \), in which the \( \omega^* = 2\pi \times 50 \).
From Figure 5, we can see that transient magnitude $A_n(t)$ remains the same. At the time when $t=0$, the initial phase of signal $f(t)$ is $\varphi = \frac{\pi}{6}$. And in the monotonically increasing section $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the frequency of original signal $f(t)$ is lower than the frequency of $f^*(t)$. So for low-frequency steady state signal, generalized sinusoidal transform method can extract signal’s primitive characters.

3.1.4. Decaying DC signal

First we set the original signal as $f(t) = e^{-\frac{t}{0.1}}$, $\omega = 2\pi \times 50$, $f_s = 10kHz$, and the sampling window time is 20ms. We know that the signal will turn into the form of $f^*(t) = A_n(t)\sin[\omega^* t + \varphi(t)]$, in which the $\omega^* = 2\pi \times 50$. 

Figure 5. Diagram of low-frequency steady state signal
From Figure 6, we can see the trend of transient magnitude $A_n(t)$ is consistent with $f(t)$. And when $t=0$, the initial phase $\varphi(t)$ monotonically decreases within the section $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the frequency of original signal $f(t)$ is lower than the frequency of $f^*(t)$.

3.1.5. Law of the transient phase $\varphi(t)$ of single-component signal

Now we can analyze the law of the transient phase $\varphi(t)$ of single-component signal. Imagine that the original signal is $f(t) = \sin(n\omega t + \phi)$, where $n$ is a random number that is above 0. After taking the Hilbert transform we can get $h(t) = H[f(t)] = \cos(n\omega t + \phi)$. In this way, the primary function is

\[
\begin{align*}
\Phi(t) &= \sin(n\omega t + \phi) \cos \omega t + \cos(n\omega t + \phi) \sin \omega t \\
\Psi(t) &= \sin(n\omega t + \phi) \sin \omega t - \cos(n\omega t + \phi) \cos \omega t
\end{align*}
\]

(25)

To put it into another way...
\[
\begin{align*}
\Phi(t) &= \sin[(n-1)\omega t + \phi] \\
\Psi(t) &= \cos[(n-1)\omega t + \phi]
\end{align*}
\]  

Then

\[
\frac{A(t)}{B(t)} = \tan[(n-1)\omega t + \phi]
\]

and

\[
\varphi(t) = \arctan\frac{A(t)}{B(t)} = \text{mod}[(n-1)\omega t + \phi, \pi]
\]

Hence, if we generalize the conclusion within the range of \((-\frac{\pi}{2}, \frac{\pi}{2})\), we can see that transient phase is changing with the change of angular velocity \((n-1)\omega\).

Figure 7. Illustrated chart of instantaneous phase
From Figure 7, we can conclude that when \( n-1 > 0 \), and the frequency of original signal \( f(t) \) is bigger, transient phase \( \varphi(t) \) changes faster. And for decaying DC signal, it can be seen as a special situation of \( n=0 \).

3.2. Combined Signal

In practical power systems, the power-frequency steady-state signal and decaying DC signal always come together. In the electromagnetic transient status, signal energy will be embedded in this combined signal. The three-phase short circuit currents in the high power supply system can be written as follows:

\[
i_f(t) = I_m \sin(\omega t + \phi) + [I_m(\sin \phi) - I_m \sin \phi]e^{-t/\tau}
\]

(29)

In this part, we will take an example to illustrate the correctness of generalized sinusoidal transform method. Here choose \( f = \sin(100\pi t + \pi/4) + e^{-t/0.03} \) as the original signal.

![Figure 8. Diagram of Combined Signal](image_url)
From Figure 8, we can see that the law of transient magnitude $A_m(t)$ is in correspondence with original signal $f(t)$. The transient phase fluctuates a lot at the beginning because of decaying DC signal. When the DC signal decreases to zero, there is only power-frequency steady-state signal, and the transient phase is $\phi(t) = 45^\circ$.

Here we will also talk about one of the common signal that usually occurs during the electromagnetic transient process; this is power-frequency steady-state signal and decaying high-frequency signal. In this part, we will take an example to illustrate the correctness of generalized sinusoidal transform method. Here I choose $f = \sin(100\pi t + \frac{\pi}{4}) + 0.5 \sin(300\pi t) e^{-0.02}$ as the original signal.

![Graphs showing signal decomposition and transformation](image)

Figure 9. Diagram of power frequency steady state with decaying high frequency signal

From Figure 9, we can see that high frequency signals usually exist in the first two circles. When high frequency signals decays to zero, transient magnitude $A_m(t)$ and transient phase $\phi(t)$ will come to their steady state.
3.3. Complex Signal

In this part, we will consider the following signal $f(t)$ as an example to illustrate the correctness of generalized sinusoidal transform method.

$$
f(t) = \begin{cases} 
10\sin(50 \times 2\pi t + \frac{\pi}{2}) , & 0 \leq t \leq 0.02s \\
10e^{0.05} \sin(50 \times 2\pi t + \frac{\pi}{2}) , & 0.02 \leq t \leq 0.042s \\
10e^{0.1} \sin(50 \times 2\pi t + \frac{\pi}{6}) , & 0.042 \leq t \leq 0.1s 
\end{cases}
$$

![Diagram of intricate signal](image)

Figure 10. Diagram of intricate signal

From Figure 10, we can see that the generalized sinusoidal function $f^*(t)$ can represent the original function well, and can show people the trend of changes of transient magnitude $A_m(t)$ and transient phase $\phi(t)$. 
This new method, generalized sinusoidal transform method brings forward a new viewpoint to define transient magnitude and transient phase. In this chapter, we explained this method in a mathematical way and expanded the definition from steady state magnitude and phase to transient state. This can be a foundation for the following chapter to discuss of the engineering application.
CHAPTER 4
ENGINEERING APPLICATION OF GENERALIZED SINUSOIDAL TRANSFORMATION METHOD

The new generalized sinusoidal transform method has a broad application space in distribution systems. In this chapter, I will show the basic use of this method in the most commonly used faulted system.

4.1. Equivalent Network and Analysis of Zero Sequence Current

In three-phase system, line coupling makes it difficult to analyze the transient state of single-phase earth fault. So we need to do the transform to help analyze.

4.1.1. Zero sequence current

Below are the steps in the analysis method:

1. Analyze of the current

The capacitance to ground in each phase is equal to each other when the system works in normal condition, and the current will be ahead of voltage in phase. When single-phase earth fault happens, the voltage of fault position can suddenly drop to 0, while the other two phases' voltage can increase to $\sqrt{3}$ of the original voltage.

Analyzing the status of the steady state of the single-phase earth fault, the current of the healthy feeder is the capacitance current, it flows from the buses to lines. On the contrary, the direction of fault feeder is from lines to the buses.

2. Phase-model transformation
Line coupling makes it difficult to analyze the transient status of single-phase earth fault in the triphase system. We need to transform the phase model to a system without electromagnetic coupling. In this project, we will use Karrenbauer transform\cite{8}. The transfer matrix is:

\[
T_m = \begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2 \\
\end{bmatrix}
\]

The inverse of the matrix is:

\[
T_m^{-1} = \begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2 \\
\end{bmatrix}
\]

Then we come up with a simple network as shown below in Figure 11.

Figure 11. Combined modal network of single phase earthed fault

In Figure 11, \(U_f\) is a negative value of the fault point under formal working condition, and works as the exciting source. Karrenbauer transform enjoys the same electric
parameters and physically means with zero-sequence model, and in the following analysis, the two models are seen to be the same.

4.1.2. Arc-suppression-coil-ground neutral system

The equivalent network is seen as follows in Figure 12.

In Figure 12, the inductive current goes through the Arc-suppression-coil, and the capacitive current goes through the lines. When the system come into steady state after single phase to ground fault, the voltage in the system will remains stable, and the current between phases is small enough to be ignored.

Inductive current will go back to the neutral point through the fault line, offset the capacitive current to decrease the fault current. There are three ways to compensate according to the capacitance current compensation:
1. Complete compensation

\[ I_L = I_{CE} \]. Current at the fault point is zero. When system works under this condition, series resonance may cause resonant overvoltage that is harmful to the system. In practical applications, it is seldom used.

2. Under-compensation

\[ I_L < I_{CE} \]. Current at the fault point is capacitive. When the system operating mode changes, there is a chance to go back to complete compensation. In practical application, it is seldom used either.

3. Over-compensation

\[ I_L > I_{CE} \]. Current at the fault point is inductive. This is the most widely used way in power system management.

If the arc-suppression-coil-ground neutral system has more than one branch, the structure of it can be seen as a \( \Pi \) model, and the arc-suppression-coil can be substitute by a series \( L \) and \( R \) circuit. When the single-phase earth fault happens, the zero sequence network can be seen as follows (the left structure in Figure 13). In reality, zero sequence \( R \) is far less from zero sequence capacitance, so we usually ignore it. Thus the system can be changed to the right structure in Figure 13.
4.1.3. Transient zero sequence current

Arc-suppression-coil-ground neutral system is most popular in industry. In theory, the coil can operate under-complete compensation status and over-compensation status. When the system comes with single-phase earth fault, the coil can produce inductive current to compensate capacitive current. Transient zero sequence current has a very representative characteristic, and content below will show it.

In the following research, we will take zero sequence voltage as reference. We can see that zero sequence current in the fault line will change its phase. Before the single-phase earth fault happens, three phase voltages are balanced, the zero sequence current in each line flow from bus to line and inductive current in the arc coil is very small.
Figure 14. Directions of zero-sequence current before fault

Figure 15. Directions of zero-sequence current at fault instant

Figure 16. Directions of zero-sequence current at fault steady state
When single-phase earth fault happens, the compensation from the coil can not change in a short time (it remains the same of the normal state in Figure 14), so the compensation function is not so obvious. We take the zero sequence voltage of the bus as the reference, and the zero sequence current is a reactive current that flows from the line to the bus in the fault line (shown in Figure 15), while the current in the healthy lines is opposite to the fault one. As time moved on, the compensation function has bigger influence on the system. Inductive current increases and zero sequence current in the fault line decreases, finally it reaches the steady state and the system will be similar to Figure 16.

4.1.4 Simulation of fault line

Now we can use Simulink to simulate the transient zero sequence current, as shown below in Figure 17.
In Figure 17, line 1, line 2 and line 3 are all directly link to the bus. To analyze the line, we can use pi model (line 4). A three phase fault happens at line 3 and we can measure each value with the help of Figure 18, including transient current and transient voltage. The results are shown below in Figure 19.
From Figure 19, we can see that the active line is the current of fault line while the dotted line is the current of healthy line. From the analysis above, we can conclude the following characteristics of the zero sequence current in the arc-suppression-coil-ground neutral system;

1. The transient fault current in the fault line has opposite flow direction to healthy lines. And the fault current magnitude nearly equals to the sum of the healthy line current. That is to say the fault current magnitude is the largest\textsuperscript{[9]}

2. During the steady state of fault, the fault current is obviously smaller than the transient status. If the coil is changed to over-compensation, the current in the fault line will eventually be in the same phase with healthy lines.

Comparing with ground neutral system, current in the non-ground neutral system will not change its phase. That is to say, current in the fault line will always keep opposite phase with healthy lines. Based on the arc-suppression-coil-ground neutral system, if we can
make good use of the changes of current (magnitude and phase), and extract the signals properly, then we can analyze the fault in a more effective way.

4.2. Faulty Line Detection Method Based on the New Method

A synergetic theory based faulty line detection method for single-phase to ground fault occurred in small current neutral grounding system is proposed in this section.

Figure 20 below shows a distributed parameter model based on MATLAB/Simulink. The transformer is 110/10.5 kV, and transformer capacity is 50MVA.

There are two ways to distinguish between the healthy line and the fault line, which are explained below:

① With the help of transient current
Figure 21. Instantaneous amplitude of zero-sequence current

From Figure 21, we can see that zero current direction of healthy line and fault line is opposite when short fault happens. Also, the fault line has the largest current magnitude, and this result is consistent with theory and analysis. Current in the fault line decreases and finally stays at the steady state, but current in the healthy line changes a little. This can be the obvious proof to detect the fault line and healthy line.

2. With the help of transient phase

Figure 22. Instantaneous phase of zero-sequence current
If we want to tell the fault line from the phase of each line is also possible. Like shown in Figure 23, after the generalized sinusoidal transfer method, the fault current's direction changes while the direction of healthy lines remain the same.

4.3. Other Applications in the Distribution Systems

To calculate the fault location is one of the main directions in power system analysis. Nowadays, the mature method is usually put into use in transmission system. With the increase of need, people are badly in need of an effective method to solve the problem in distribution system. If we try this generalized sinusoidal method to the system, we can get the current we want and help calculate the definite distance of fault.
CHAPTER 5
CONCLUSIONS AND FUTURE WORK

As the continuation of the previous work\cite{2,7,9}, this project presents a new method to analyze signals in power systems by time domain, generalized sinusoidal transform method. The new method is developed on the basis of the Hilbert transform method. First step of using this new method is to build a couple of primary functions, then use primary functions to express the generalized sinusoidal function. In this project, we try to deduce the method with mathematical theory and then give the important characteristics in detail. This new method adopts a new kind of angle to generalize magnitude and phase from steady state concept to transient domain. In this way, transient magnitude and transient phase can help analyze signals' characters in a more accurate way. Then detailed research has been made on the typical signals in the power system using this generalized sinusoidal transform method. From the results of simulation, we can see that this new method can catch signal's detailed changes by using the concept of transient magnitude and transient phase. This advantage is superior in comparison to traditional frequency domain and time domain analysis. Finally, we have listed several kinds of application in engineering field by using this generalized sinusoidal transform method to help chose fault lines or to find out fault location. Topics included are arc-suppression-coil-ground neutral system, including changes of zero sequence current and fault line transient magnitude and transient phase.

Everything has its own limits. In this project, we have evolved the generalized sinusoidal transfer method under theoretical analysis and have done further research under ideal
condition. While the simulation results can represent the reality to some degree, when it comes to engineering aspect, there is still a long way to go. In this project, the generalized sinusoidal transform method is used to analyze discrete signals instead of any type of signal, so future work is to find another form to help analyze continuous signal. What is more, this method is almost done on the basis of theoretical discussion, how it works under the practical condition is still unknown. For the new developing method, how to put it into effect in different fields, such as communication engineering, biomedical research and geopolitical investigation still needs long-term work.
REFERENCES


