CENTRIFUGAL FORCE PROPULSION ENGINE

A Thesis

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Peter Hoang Nguyen

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CENTRIFUGAL FORCE PROPULSION ENGINE

A Thesis

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Department of Mechanical Engineering
Abstract

of

CENTRIFUGAL FORCE PROPULSION ENGINE

by

Peter Hoang Nguyen

In contrast to a current propellant-based rocket, the concept in this thesis uses the rotational mass to create an imbalanced centrifugal force in a non-circular enclosure, which results in a propulsion force being generated. The approach to analyze and validate this imbalance centrifugal force using the simple harmonic motion method through the use of Mathematica software to solve and plot out the cam profile curve, the velocity, the acceleration, the centrifugal force, and the linear momentum. This thesis would be an ideal solution to space propulsion. Furthermore, this concept is lightweight, especially compared to expellants, because the necessary thrust is self-contained, repeated, and reused.

_________________________, Committee Chair
Dr. Akihiko Kumagai

_________________________
Date
DEDICATION

This work is dedicated to my parents, my wife, and my three children whom had endured through all the hardships and still encouraged me to pursue my dream and research.
ACKNOWLEDGMENTS

This thesis would have remained a dream had it not been for my Master Thesis Advisor. It is with immense gratitude that I acknowledge the support and the help of my Master Thesis Advisor, Dr. Akihiko Kumagai.

It also gives me great pleasure in acknowledging the support and the help of Professor Dr. Ilhan Tuzcu, my second advisor, whom has spent his invaluable time to read and review my thesis.
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1 Chapter 1
INTRODUCTION

1.1 Problem Statement

Most spaceships today use propellant-based rocket engines. A rocket requires expulsion of mass to move the rocket forward; this expulsion will generate a great amount of thrust to move the rocket forward. The amount of fuel the rocket carries limits the thrust, and fuel adds to the weight of the whole rocket. Because the rocket exhausts the fuel supply, there is a limit to its speed.

Consider a rock attached to one end of a rope twirling around in circular motion then being released to provide a linear motion. The question is, “how can we harness the rotational motion of an object to provide a linear motion to another object?”

1.2 Proposed Concept Solution 1

Suppose a ball rotating around a non-circular smooth closed Enclosure with the help of the Rotating Fork as Figure 1.2.1 illustrates. To analyze further, Figure 1.2.2 demonstrates, a ball starts at 0 degrees and rotates counter clockwise with an angle $\alpha_1$ and a radius $R_1$. The ball continues to angle $\beta_1$ with a transition radius between $R_1$ and $R_2$. At the end of angle $\beta_1$ and continuing with angle $\alpha_2$ to the beginning of angle $\beta_2$, the ball rotates with radius $R_2$. Then at angle $\beta_2$, the ball is transitioning the radius from $R_2$ to $R_1$. Finally, the ball completes one cycle at angle $\alpha_3$ with the radius $R_1$. 
Figure 1.2.1 One Ball Rotating
Figure 1.2.2 Profile Path Regions
1.3 Proposed Concept Solution 2

A ball rotating by itself would not be an effective way to demonstrate a linear motion because it would wobble all around. An addition of another ball opposite from the first ball by 180 degrees would eliminate the wobbling effect as Figure 1.2.3 clearly demonstrates. That is, the two balls at 180 degrees from each other both having a radius of R1 would cancel each other’s force. Moreover, when one ball is at R2 and the other ball is at R1, there would be a noticeable net force between the two balls.

Figure 1.2.3 Two Balls Rotating
1.4 Benefits of the Concept

- The primary objective of the proposed concept is to provide a directional force through a sealed circulating mass.

- Another objective of the concept is to provide vehicle maneuverability in three dimensions e.i. when the engine net force is directed at zero degree and the require net force is now needed in the 90 degrees direction, the problem could be solved by rotating the engine by 90 degrees.

- A further objective of the concept is to reduce the overall weight of a space vehicle.
Chapter 2

BACKGROUND OF THE STUDY

This concept is similar to the cam-follower design concept. Therefore, the equations of motion were derived similarly to a cam-follower design. In one cycle, which is a full circle, the division in regions is necessary to separate and quantify the equations of motion. As mentioned in Figure 1.2.1, there are a total of five regions, Region A, B, C, D, and E. the radii for Regions A and E are the same. Region C is similar to regions A and E, but the radius is larger than that at Regions A and E. Finally, the simple harmonic motion profile for regions B (rise) and D (return).

2.1 Dividing Regions and Associating Equations of Motions to Each Region

Defining constants:

\[
\begin{align*}
\alpha_1 &= \frac{3\pi}{4} & \beta_1 &= \frac{\pi}{8} & R_2 &= 0.0889 \text{ m} \\
\beta_2 &= \frac{\pi}{8} & \alpha_3 &= \frac{3\pi}{4} & m_{\text{ball}} &= 0.1 \text{ kg} \\
\alpha_2 &= \frac{\pi}{4} & R_1 &= 0.0762 \text{ m} & m_{\text{total2}} &= 0.427 \text{ kg} \\
\omega &= 1000 \text{ rev/min} & \quad \text{or} & \quad \omega &= 104.7 \text{ rad/s}
\end{align*}
\]

The mass that is used in this thesis is 0.1 kg round stainless steel ball plus 0.227 kg for the Enclosure and the Rotating Fork of the engine. The total mass of the one ball engine is about 0.327 kg, not including the motor and the energy source require to power the engine. And the total mass of two balls engine, \( m_{\text{total2}} \), is about 0.427 kg.
Region A ($0 < \theta < \alpha 1$)
\[
    r = R_1
\]  
(1)
\[
    \dot{r} = 0
\]  
(2)
\[
    \ddot{r} = 0
\]  
(3)
\[
    \dot{\theta} = \omega
\]  
(4)
\[
    \ddot{\theta} = 0
\]  
(5)
\[
    v_r = \dot{r}
\]  
(6)

Substituting equation (2) into equation (6)
\[
    v_r = 0
\]  
(7)
\[
    v_\theta = r \dot{\theta}
\]  
(8)

Substituting equation (4) into equation (8)
\[
    v_\theta = r \omega
\]  
(9)
\[
    v_x = v_r \cos(\theta) + v_\theta \cos\left(\theta + \frac{\pi}{2}\right)
\]  
(10)

Substituting equation (7) and (9) into equation (10)
\[
    v_x = r \omega \cos\left(\theta + \frac{\pi}{2}\right)
\]  
(11)
\[
    v_y = v_r \sin(\theta) + v_\theta \sin\left(\theta + \frac{\pi}{2}\right)
\]  
(12)

Substituting equation (7) and (9) into equation (12)
\[
    v_y = r \omega \sin\left(\theta + \frac{\pi}{2}\right)
\]  
(13)
\[
    a_r = \ddot{r} + r (\dot{\theta})^2
\]  
(14)
Substituting equation (1) (2) and (4) into equation (14)

\[ a_r = r \omega^2 \]  
\[ a_\theta = r \ddot{\theta} + 2 r \dot{\theta} \]  
\[ a_x = a_r \cos \theta + a_\theta \cos \left( \theta + \frac{\pi}{2} \right) \]  
Substituting equation (15) and (17) into equation (18)

\[ a_x = r \omega^2 \cos \theta \]  
\[ a_y = a_r \sin \theta + a_\theta \sin \left( \theta + \frac{\pi}{2} \right) \]  
Substituting equation (15) and (17) into equation (20)

\[ a_y = r \omega^2 \sin \theta \]  
\[ F_x = m_{ball} a_x \]  
Substituting equation (19) into equation (22)

\[ F_x = m_{ball} r \omega^2 \cos \theta \]  
\[ F_y = m_{ball} a_y \]  
Substituting equation (21) into equation (23)

\[ F_y = m_{ball} r \omega^2 \sin \theta \]  
Region B (\( a_1 < \theta \leq \beta_1 \))

\[ r = R_1 + \frac{1}{2} (R_2 - R_1) \left( 1 - \cos \left[ \frac{\pi (\theta - \alpha_1)}{\beta_1} \right] \right) \]
Let
\[ \phi = \frac{\pi(\theta - \alpha_1)}{\beta_1} \]

then \[ \frac{d}{dt}(\cos \phi) = -\frac{\dot{\phi}}{\dot{\phi}} \sin \phi = -\frac{\pi}{\beta_1} \dot{\phi} \sin \phi = -\frac{\pi}{\beta_1} \omega \sin \phi \] \hspace{1cm} (27)

\[ \dot{r} = \frac{d}{dt}\left[R_1 + \frac{1}{2}(R_2 - R_1)(1 - \cos[\phi])\right] \] \hspace{1cm} (28)

Taking first derivative then substituting equation (27) into equation (28)

\[ \dot{r} = \frac{\pi(R_2 - R_1)}{2\beta_1} \omega \sin \left[\frac{\pi(\theta - \alpha_1)}{\beta_1}\right] \] \hspace{1cm} (29)

\[ \ddot{r} = \frac{d^2}{dt^2}\left[R_1 + \frac{1}{2}(R_2 - R_1)(1 - \cos[\phi])\right] \] \hspace{1cm} (30)

Taking second derivative then substituting equation (27) into equation (30)

\[ \ddot{r} = \frac{\pi^2(R_2 - R_1)}{2\beta_1^2} \omega^2 \cos \left[\frac{\pi(\theta - \alpha_1)}{\beta_1}\right] \] \hspace{1cm} (31)

Substituting equation (29) into equation (6)

\[ v_r = \frac{\pi(R_2 - R_1)}{2\beta_1} \omega \sin \left[\frac{\pi(\theta - \alpha_1)}{\beta_1}\right] \] \hspace{1cm} (32)

Substituting equation (4) and (26) into equation (8)

\[ v_\theta = \left(R_1 + \frac{1}{2}(R_2 - R_1)(1 - \cos\left[\frac{\pi(\theta - \alpha_1)}{\beta_1}\right])\right)\omega \] \hspace{1cm} (33)

Substituting equation (32) and (33) into equation (10)

\[ v_x = \frac{\pi(R_2 - R_1)}{2\beta_1} \omega \sin \left[\frac{\pi(\theta - \alpha_1)}{\beta_1}\right] \cos(\theta) \]

\[ + \left(R_1 + \frac{1}{2}(R_2 - R_1)(1 - \cos\left[\frac{\pi(\theta - \alpha_1)}{\beta_1}\right])\right)\omega \cos(\theta + \frac{\pi}{2}) \] \hspace{1cm} (34)
Substituting equation (32) and (33) into equation (12)

\[ v_y = \frac{\pi(R_2-R_1)}{2\beta_1} \omega \sin \left[ \frac{\pi(\theta-a_2)}{\beta_1} \right] \sin(\theta) \]

\[ + \left( R_1 + \frac{1}{2} (R_2 - R_1) \left( 1 - \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \right) \right) \omega \sin(\theta + \frac{\pi}{2}) \]  \( (35) \)

Substituting equation (4) (26) and (31) into equation (14)

\[ a_r = \frac{\pi^2(R_2-R_1)}{2\beta_1^2} \omega^2 \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] + \left( R_1 + \frac{1}{2} (R_2 - R_1) \left( 1 - \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \right) \right) \omega^2 \]  \( (36) \)

Substituting equation (4) (5) (26) and (29) into equation (16) and simplify

\[ a_\theta = \frac{\pi(R_2-R_1)}{\beta_1} \omega^2 \sin \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \]  \( (37) \)

Substituting equation (36) and (37) into equation (18)

\[ a_x = \]

\[ \left( \frac{\pi^2(R_2-R_1)}{2\beta_1^2} \omega^2 \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] + \left( R_1 + \frac{1}{2} (R_2 - R_1) \left( 1 - \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \right) \right) \omega^2 \right) \cos \theta \]

\[ + \frac{\pi(R_2-R_1)}{\beta_1} \omega^2 \sin \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \cos \left( \theta + \frac{\pi}{2} \right) \]  \( (38) \)

Substituting equation (36) and (37) into equation (20)

\[ a_y = \left( \frac{\pi^2(R_2-R_1)}{2\beta_1^2} \omega^2 \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] + \left( R_1 + \frac{1}{2} (R_2 - R_1) \left( 1 - \cos \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \right) \right) \omega^2 \right) \sin \theta \]

\[ + \frac{\pi(R_2-R_1)}{\beta_1} \omega^2 \sin \left[ \frac{\pi(\theta-a_1)}{\beta_1} \right] \sin \left( \theta + \frac{\pi}{2} \right) \]  \( (39) \)
Substituting equation (36) and (37) into equation (22)

\[ F_x = \]

\[ m_{batl} \left( \frac{\pi^2(R_2-R_1)}{2\beta_1^2} \omega^2 \cos \left[ \frac{\pi(\theta-\alpha_1)}{\beta_1} \right] + \left( R_1 + \frac{1}{2}(R_2-R_1) \left( 1 - \cos \left[ \frac{\pi(\theta-\alpha_1)}{\beta_1} \right] \right) \right) \omega^2 \cos \theta + \]

\[ \frac{\pi(R_2-R_1)}{\beta_1} \omega^2 \sin \left[ \frac{\pi(\theta-\alpha_1)}{\beta_1} \right] \cos \left( \theta + \frac{\pi}{2} \right) \right) \]  

(40)

Substituting equation (36) and (37) into equation (24)

\[ F_y = \]

\[ m_{batl} \left( \frac{\pi^2(R_2-R_1)}{2\beta_1^2} \omega^2 \cos \left[ \frac{\pi(\theta-\alpha_1)}{\beta_1} \right] + \left( R_1 + \frac{1}{2}(R_2-R_1) \left( 1 - \cos \left[ \frac{\pi(\theta-\alpha_1)}{\beta_1} \right] \right) \right) \omega^2 \sin \theta + \]

\[ \frac{\pi(R_2-R_1)}{\beta_1} \omega^2 \sin \left[ \frac{\pi(\theta-\alpha_1)}{\beta_1} \right] \sin \left( \theta + \frac{\pi}{2} \right) \right) \]  

(41)

Region C \((\alpha_1 + \beta_1 < \theta \leq \alpha_2)\)

Since Region C is similar to Region A except the radius, reapply equations (2) to (25) here with the radius of \( r = R_2 \).

Region D \((\alpha_1 + \beta_1 + \alpha_2 < \theta \leq \beta_2)\)

\[ r = R_1 + \frac{1}{2}(R_2-R_1) (1 + \cos \left[ \frac{\pi(\theta-(\alpha_1 + \beta_1 + \alpha_2))}{\beta_2} \right] \]

and let \( \phi = \frac{\pi(\theta-(\alpha_1 + \beta_1 + \alpha_2))}{\beta_2} \)

\[ \frac{d}{dt} (\cos \phi) = -\dot{\phi} \sin \phi = -\frac{\pi}{\beta_2} \dot{\phi} \sin \phi = -\frac{\pi}{\beta_2} \omega \sin \phi \]
\[
\dot{r} = \frac{d}{dt} \left[ R_1 \frac{1}{2} (R_2 - R_1)(1 + \cos[\theta]) \right] = -\frac{\pi (R_2 - R_1)}{2\beta_2} \omega \sin \left( \frac{\pi (\theta - (\alpha_1 + \beta_1 + \alpha_2))}{\beta_2} \right) \tag{43}
\]
\[
\ddot{r} = \frac{d^2}{dt^2} \left[ R_1 \frac{1}{2} (R_2 - R_1)(1 + \cos[\theta]) \right] = -\frac{\pi^2 (R_2 - R_1)}{2\beta_2^2} \omega^2 \cos \left( \frac{\pi (\theta - (\alpha_1 + \beta_1 + \alpha_2))}{\beta_2} \right) \tag{44}
\]

Similarly to Region B to obtain: \(\theta', \theta'', v_r, v_\theta, v_x, v_y, a_r, a_\theta, a_x, a_y, F_x, \text{ and } F_y\)

used equations (4) (5) (6) (8) (10) (12) (14) (16) (18) (20) (22) and (24) respectively with proper substitution of equations (42) (43) and (44).

Region E \((\alpha 1 + \beta 1 + \alpha 2 + \beta 2 < \theta \leq 2\pi)\)

Since Region E is the same as Region A, reapply equations from (1) to (25) to this region.

2.2 Using Mathematica Software to Solve and Plot out the Result of Regions

The following graphs help describe the equations of motion in one cycle. There are profile graphs describing the path of the ball. There are velocity, acceleration, and force graphs, which break down into X and Y components. Then all the previously mentioned graphs were shifted by 180 to describe the motion of the second ball. Finally, there are combined graphs representing two balls for one cycle to demonstrate a clearer picture of the net force being generated.
Figure 2.2.1 shows the profile of the path the ball would go around in one cycle. Region A is between zero and $6\pi/8$ with the constant radius of 0.0762 m. Region B is between $6\pi/8$ and $7\pi/8$ with the radius changing from 0.0762 m to approximately 0.0889 m. Region C is between $7\pi/8$ and $9\pi/8$ with a constant radius of approximately 0.0889 m. Region D is between $9\pi/8$ and $10\pi/8$ with the radius constantly changing from 0.0889 m to 0.0762 m. Finally, Region E is between $10\pi/8$ and $2\pi$ with a radius of 0.0762 m.
Figure 2.2.2 shows a velocity of X component (blue) and Y component (magenta). Notice at regions B and D there is a shift curve as the ball is experiencing an increasing radius and decreasing radius, respectively. As for the X component, there is not much difference except in regions B and D, and there is a little side shifting in the curve.
Figure 2.2.3 demonstrates acceleration in the X component (blue) and Y component (magenta). There is no difference between the highs and lows for the Y component in one cycle except there is a little side shift at regions B and D. However, there is a noticeable difference in the X component between the highs and the lows of one cycle.
Since force = mass * acceleration, Figure 2.2.4 demonstrates the same shape as
the acceleration but is being multiplied by the mass to give it a force unit.

Figure 2.2.4 Force
Figure 2.2.5 plots a profile shift of 180 degrees for the second ball. That is, instead of starting at 0 degrees, the ball is starting at 180 degrees. Region C is where 180 degrees is and it is now zero radians.
Figure 2.2.6 plots the shift of 180 degrees for the velocity in the X and Y components for the second ball.
Figure 2.2.7 plots the acceleration shifted by 180 degrees for the second ball.

![Figure 2.2.7 180 Degrees Shift Acceleration](image)
Figure 2.2.8 plots the force shifted by 180 degrees for the second ball.

![Figure 2.2.8 180 Degrees Shift Force](image-url)
Figure 2.2.9 plots the two balls rotating simultaneously but 180 degrees apart. As the plot demonstrates, the forces in the Y component are nearly negated. However, for the X component, the forces are unbalanced. Therefore, the net force would be pushing the base higher on the positive Y-axis on this plot. In fact, we can roughly calculate the net force at π equals to 13 N force in the X component by the difference at ball 2 approximately equaling an 85 N force and ball 1 approximately equaling a 98 N force.
Figure 2.2.10 is a combination of the two balls. This shows there is an average net force of about 14 N for the X components and zero N for the Y component, as previously mentioned.

Figure 2.2.10 The Difference Between Two Balls Forces

\[
\begin{align*}
0 & \quad \frac{\pi}{8} & \quad \frac{2\pi}{8} & \quad \frac{6\pi}{8} & \quad \frac{7\pi}{8} & \quad \frac{9\pi}{8} & \quad \frac{10\pi}{8} & \quad \frac{14\pi}{8} & \quad \frac{15\pi}{8} & \quad \frac{2\pi}{8}
\end{align*}
\]
Chapter 3

ANALYSIS OF DATA

Analysis of each profile, velocity, acceleration, force, impulse and momentum are analyzed, providing the basis for the centrifugal force propulsion concept.

3.1 Profiles

Figure 2.2.1 for the profile shows a straight horizontal line at regions A, C and E, meaning it is constant. However, when it has to be changing from R1 to R2 and vice versa at regions B and D, it shows a positive and negative slope, respectively. Figure 2.2.5 shows the profile shifted by 180 degrees, which looks similar to Figure 2.2.1.

3.2 Velocities

With regions A and E having a constant radius of R1, the equations of motion are relatively simple. X and Y components in these regions are changing relatively equally around the circular path with regard to velocity. Figure 2.2.2 shows the velocities with regard to the X and Y components.

In regions B and D, the radius is constantly changing from R1 to R2 and R2 to R1, respectively. The equations of motion are different between regions A and E, and X and Y components are changing unequally. Figure 2.2.2 shows the X component velocity for region B is a positive slope while the Y component velocity is a negative slope.
During region B, the X component velocity changes less than the Y velocity component, and this shows up a little side shift to the plot for both X and Y component velocities. At region D, the X component velocity continues on the positive slope. The X velocity looks like it was mirrored in the X-axis and Y-axis at the same time. While the Y velocity component is only mirrored around the Y-axis at $\pi$, it reverses direction and goes to the positive slope.

Figure 2.2.2 shows region C as a normal sine and cosine curve with the X velocity component a positive slope while the Y velocity component is changing from a negative slope to a positive slope. In comparison to regions A and E, region C behaves the same except the radius is R2 instead of R1.

### 3.3 Accelerations

In Figure 1.2.1, regions A and E have a radius of constant R1; therefore, X and Y components in these regions are changing relatively equally around the circular path with regard to acceleration. Figure 2.2.3 shows the accelerations in the X and Y components at these regions.

Region C is behaving like regions A and E except the accelerations are more at region C than regions A and E because the radius R2 is greater than R1.

Regions B and D are where the radii are changing. At these locations, there is discontinuity in the graph in the beginning and ending of each region due to the fact the apparatus was abruptly changing from a circular motion to a tangential motion.
At regions B and D, there is a gap in the graph of the acceleration at the beginning and ending of each region has to be resolved. Also as the number of balls in the engine increase, the occurrence of the net acceleration happens more frequently. This means that the more balls present in the engine, the smoother the net acceleration.

3.4 Forces

Since $F = ma$, Figure 2.2.4 shows the graph behaves just like the acceleration graph with a multiplier $m$ to change the size of the graph.

Comparing Figure 2.2.4, a force of a single ball in one cycle, to Figure 2.2.9, a force of two balls in one cycle, it is clear that with a single ball there would be a lot of wobbling as there is no counter force to balance the overall force. However, as another ball was introduced to the engine, the engine produced a cleaner net force and the wobbling would be significantly minimized in one cycle.

There are many ways the net force can be increased. One way is to increase the RPM. Another way of increasing the force is to increase the radius, thus increasing the size of the engine. Another way to increase the net force is to increase the mass of the rotating ball. Finally, one could combine some or all the above ways to increase the net force.

As more balls are in the rotating circle, more net forces are occurring in a cycle, which means a smoother distribution of force. This implies a smoother experience for the
vehicle. As long as there are two or more propulsion engines in a vehicle, the net force in a direction of a 3D environment can be controlled.

3.5 Impulse and Momentum

Impulse and Momentum equation

\[ mV_1 + Imp_{1\rightarrow 2} = mV_2 \]  \hspace{1cm} (48)

The net average impulse of the x force is calculated by the area under the force graph of Figure 2.2.10 to be around 13 N in 1.2 radians for one ball.

\[ Imp_{1\rightarrow 2} = \text{Area} = 2(13 \text{ N} \times 1.2 \text{ rad}) = 31.2 \text{ N rad} \]  \hspace{1cm} (45)

Converting the time it takes for one cycle

\[ t = \left( \frac{\text{rad}}{\text{rev}} \right) \left( \frac{1}{\omega} \right) = \frac{2\pi}{\text{rev}} \left( \frac{s}{104.7 \text{ rad}} \right) = 0.06 \frac{s}{\text{rev}} \]  \hspace{1cm} (46)

or the time it takes per radian

\[ t = \frac{0.06s}{\text{rev}} \left( \frac{\text{rev}}{2\pi} \right) = 0.01 \frac{s}{\text{rad}} \]  \hspace{1cm} (47)

Since the initial velocity, \( V_1 \), is assume to be zero therefore equation (3) becomes

\[ Imp_{1\rightarrow 2} = mV_2 \]  \hspace{1cm} (49)

Impulse generated by two balls in one cycle

\[ (31.2 \text{ N rad}) \left( 0.01 \frac{s}{\text{rad}} \right) = 0.312 \text{ N s} \]  \hspace{1cm} (50)

Substituting (50) and the mass of two balls, \( m_{\text{ball}} \), into (49) and finding velocity, \( V_2 \)

\[ V_2 = \frac{Imp_{1\rightarrow 2}}{m} = \frac{0.312 \text{ N s}}{2 \times 0.1 \text{ kg}} = 1.6 \frac{m}{s} \]  \hspace{1cm} (51)

Therefore the momentum of the whole engine with the mass of \( m_{\text{total2}} \) is
Momentum = $m_{total}V_2 = 0.427 \, kg \times 1.6 \frac{m}{s} = 0.68 \, kg \frac{m}{s}$ \hfill (52)

3.6 Conclusions

Some of the findings and interpretations based on the generated mathematical model are just reasoning based on my own understanding of the concept. For proving the concept, which is to prove there would be a net force, the harmonic motion, the cam profile curve, the acceleration, the force, the impulse and momentum are clearly necessary.

The concept is a workable solution for the proposed propulsion engine. However, there are problems associated with the new concept that I believe need to be resolve in the future, such as noise, heat, and vibration. The advantage of having such a concept is to provide a directional force through its sealed circulating mass and to provide vehicle maneuverability in three dimensions by rotating the engine net force in the direction that is needed and being able to reduce the overall weight of a space vehicle.
Appendix

MATHEMATICA SOFTWARE SOLUTIONS

\[ \alpha_1 = 3 \cdot \frac{\pi}{4} \]
\[ f_1 = \frac{\pi}{8} \]
\[ \alpha_2 = \frac{\pi}{4} \]
\[ f_2 = \frac{\pi}{8} \]
\[ \alpha_3 = 3 \cdot \frac{\pi}{4} \]
\[ r_1 = 0.0762 \]
\[ r_2 = 0.0889 \]
\[ \omega = 104.7 \]
\[ m = 1 \]
\[ \theta^0 = \omega \]
\[ \theta^{-} = 0 \]
\[ \frac{3\pi}{4} \]
\[ \pi \]
\[ \frac{\pi}{8} \]
\[ \frac{\pi}{4} \]
\[ \frac{\pi}{8} \]
\[ \frac{3\pi}{4} \]

0.0762

0.0889

104.7

0.1

104.7

0

Region A (0 \leq \theta < \alpha_1)

\[ r[\theta] := (\theta \geq 0 \&\& \theta < \alpha_1) \]
\[ r[\theta] := r_1 \]
\[ r^\prime[\theta] \]
\[ r^\prime\prime[\theta] \]

0

0

ProfileA = Plot[r[\theta], \{\theta, 0, \frac{3\pi}{4}\}, Frame \to True, FrameLabel \to \{"Radians", "Radius (m)"\}, PlotLabel \to "Region A", LabelStyle \to Orange]
\[ v_r[\theta] := r'[\theta] \]
\[ v_\theta[\theta] := r[\theta] \times (\theta'') \]
\[ v_\phi[\theta] := v_r[\theta] \cos[\theta] + v_\theta[\theta] \cos[\theta + \frac{\pi}{2}] \]
\[ v_z[\theta] := v_r[\theta] \sin[\theta] + v_\theta[\theta] \sin[\theta + \frac{\pi}{2}] \]

0

7.97814

\[-7.97814 \sin[\theta]\]

7.97814 \cos[\theta]

VelocityA = Plot[{v_x, v_y}, {\theta, 0, \frac{3\pi}{4}}, Frame -> True, FrameLabel -> {"Radiant", "Velocity (m/s)"}, PlotLabel -> "Region A", LabelStyle -> Orange]

\[ a_r[\theta] := r''[\theta] - r[\theta] \times (\theta'')^2 \]
\[ a_\theta[\theta] := r[\theta] \times (\theta''^2) + 2 \times r'[\theta] \times (\theta'') \]
\[ a_\phi[\theta] := 0 \]

\[-835.311 \]
\[
\begin{align*}
    a_x &= a_1 \theta \cos \theta + a_0 \theta \cos \left( \theta + \frac{\pi}{2} \right) \\
    a_y &= a_1 \theta \sin \theta + a_0 \theta \sin \left( \theta + \frac{\pi}{2} \right)
\end{align*}
\]

\[
\begin{align*}
    F_{x1} &= m \ a_x \\
    F_{y1} &= m \ a_y
\end{align*}
\]

\[
\begin{align*}
    0.1 (0. - 835.311 \cos \theta) \\
    0.1 (0. - 835.311 \sin \theta)
\end{align*}
\]

AccelerationA = Plot \[ \left[ a_x, a_y \right], \left\{ \theta, 0, \frac{3\pi}{4} \right\}, \text{Frame} \rightarrow \text{True}, \]

FrameLabel \rightarrow \{"Radians", "Acceleration (m/s^2)"\}, PlotLabel \rightarrow "Region A", LabelStyle \rightarrow \text{Orange} \]

\[
\begin{align*}
    
\end{align*}
\]

FrameLabel \rightarrow \{"Radians", "Force (N)"\}, PlotLabel \rightarrow "Region A", LabelStyle \rightarrow \text{Orange} \]

Region B (\( \theta_1 < \theta < \theta_1 + \beta_1 \))

\[
\begin{align*}
    r(0.) &:= (\theta \geq \theta_1 \&\& \theta < \theta_1 + \beta_1) \\
    r(\theta) &:= 0.5 \left( r_2 - r_1 \right) (1 - \cos \left( \theta - \theta_1 \right) / \beta_1) + r_1
\end{align*}
\]
\[ 0.0762 + 0.00635 \left(1 - \cos\left(\frac{3\pi}{4} + \theta\right)\right) \]

\[ r^*[\theta] = 0.0508 \sin\left(\frac{3\pi}{4} + \theta\right) \]

\[ r^*[\theta] = 0.0064 \cos\left(\frac{3\pi}{4} + \theta\right) \]

\[ \text{ProfileB} = \text{Plot}\left[r[\theta], \{\theta, \frac{6\pi}{8}, \frac{7\pi}{8}\}, \text{Frame} \to \text{True}, \right. \]

\[ \left. \text{FrameLabel} \to \{\text{"Radians"}, \text{"Radius (m)"}\}, \text{PlotLabel} \to \text{"Region B"}, \text{LabelStyle} \to \text{Red}\right] \]

\[ v_r[\theta] := r'[\theta] \]

\[ v_r[\theta] = r[\theta] \cdot \langle \theta' \rangle \]

\[ v_\theta[\theta] := r[\theta] \cdot \langle \theta' \rangle \cos[\theta + \frac{\pi}{2}] \]

\[ v_\theta[\theta] = v_r[\theta] \cos[\theta] + v_\phi[\theta] \cos[\theta + \frac{\pi}{2}] \]

\[ v_\phi[\theta] := r[\theta] \cdot \langle \theta' \rangle \sin[\theta + \frac{\pi}{2}] \]

\[ v_\phi[\theta] = v_r[\theta] \sin[\theta] + v_\phi[\theta] \sin[\theta + \frac{\pi}{2}] \]

\[ 0.0508 \sin\left(\frac{3\pi}{4} + \theta\right) \]

\[ 104.7 \left(0.0762 + 0.00635 \left(1 - \cos\left(\frac{3\pi}{4} + \theta\right)\right)\right) \]

\[ -104.7 \left(0.0762 + 0.00635 \left(1 - \cos\left(\frac{3\pi}{4} + \theta\right)\right)\right) \sin[\theta] + 0.0508 \cos[\theta] \sin\left(\frac{3\pi}{4} + \theta\right) \]

\[ 104.7 \cos[\theta] \left(0.0762 + 0.00635 \left(1 - \cos\left(\frac{3\pi}{4} + \theta\right)\right)\right) \sin[\theta] + 0.0508 \sin[\theta] \sin\left(\frac{3\pi}{4} + \theta\right) \]

\[ \text{VelocityB} = \text{Plot}\left[v_r, v_\theta, \{\theta, \frac{6\pi}{8}, \frac{7\pi}{8}\}, \text{Frame} \to \text{True}, \right. \]

\[ \left. \text{FrameLabel} \to \{\text{"Radians"}, \text{"Velocity (m/s)"}\}, \text{PlotLabel} \to \text{"Region B"}, \text{LabelStyle} \to \text{Red}\right] \]
\[ a_{1}[\theta] := r^{*}[\theta] - r[\theta] + (\theta')^2 \]
\[ a_{2}[\theta] := r[\theta] + (\theta'') + 2r'[\theta] + (\theta')^2 \]

\[ a_{2}[\theta] = -10.9621 \left( 0.0762 + 0.00635 \left[ 1 - \cos\left( -\frac{3\pi}{4} + \theta \right) \right] + 0.4064 \cos\left( \frac{3\pi}{4} + \theta \right) \right) + 10.6375 \sin\left( \frac{3\pi}{4} + \theta \right) \]

\[ a_{3} = a_{1}[\theta] \cos[\theta] + a_{2}[\theta] \cos\left( \theta + \frac{\pi}{2} \right) \]

\[ a_{3} = a_{1}[\theta] \sin[\theta] + a_{2}[\theta] \sin\left( \theta + \frac{\pi}{2} \right) \]

\[ F_{x2} = m \, a_{3} \]
\[ F_{y2} = m \, a_{3} \]

\[ \cos[\theta] = -10.9621 \left( 0.0762 + 0.00635 \left[ 1 - \cos\left( -\frac{3\pi}{4} + \theta \right) \right] + 0.4064 \cos\left( \frac{3\pi}{4} + \theta \right) \right) - 10.6375 \sin[\theta] \sin\left( \frac{3\pi}{4} + \theta \right) \]

\[ -0.1 \left( \cos[\theta] - 10.9621 \left( 0.0762 + 0.00635 \left[ 1 - \cos\left( -\frac{3\pi}{4} + \theta \right) \right] + 0.4064 \cos\left( \frac{3\pi}{4} + \theta \right) \right) + 10.6375 \sin[\theta] \sin\left( \frac{3\pi}{4} + \theta \right) \right) \]

\[ 0.1 \left( \left( -10.9621 \left( 0.0762 + 0.00635 \left[ 1 - \cos\left( -\frac{3\pi}{4} + \theta \right) \right] + 0.4064 \cos\left( \frac{3\pi}{4} + \theta \right) \right) - 10.6375 \sin[\theta] \sin\left( \frac{3\pi}{4} + \theta \right) \right) \]

\[ \text{Acceleration} \text{B} = \text{Plot}\left[ \{ a_{x}, a_{y} \}, \left\{ \theta, \frac{6\pi}{8}, \frac{7\pi}{8} \right\} \right], \text{Frame} \to \text{True}, \]

\[ \text{FrameLabel} \to \{ "Radians", "Acceleration (m/s^2)" \}, \text{PlotLabel} \to \"Region B\" \}, \text{LabelStyle} \to \text{Red} \]
Region C \( (\alpha_1 + \beta_1 \leq \theta \leq \alpha_1 + \beta_1 + \alpha_2) \)

\[
\tau(\theta) := \begin{cases} 
\alpha_1 + \beta_1 & \text{if } \theta = \alpha_1 + \beta_1 \\
\tau(\theta) & \text{if } \theta < \alpha_1 + \beta_1 + \alpha_2
\end{cases}
\]

\[
\tau'[	heta] := r_2
\]

\[
\tau''[\theta]
\]

ProfileC = Plot\( \tau(\theta), \{\theta, \frac{6\pi}{8}, \frac{9\pi}{8}\} \), Frame → True,

FrameLabel → {"Radius (m)"}, PlotLabel → "Region C", LabelStyle → Magenta
\begin{align*}
\nu_1(\theta) & := r'[\theta] \\
\nu_2(\theta) & := r(\theta) \\
\nu_3(\theta) & := r(\theta) \times (\theta') \\
\nu_4(\theta) & := r(\theta) \cos \left( \theta + \frac{\pi}{2} \right) \\
\nu_5 & := \nu_1(\theta) \cos \theta + \nu_3(\theta) \cos \left( \theta + \frac{\pi}{2} \right) \\
\nu_6 & := \nu_1(\theta) \sin \theta + \nu_3(\theta) \sin \left( \theta + \frac{\pi}{2} \right) \\
\nu_7 & := 0 \\
9.3073 & \quad \text{Sin}[\theta] \\
9.3073 & \quad \text{Cos}[\theta] \\
\text{VelocityC} & := \text{Plot}[\{\nu_5, \nu_6\}, \{\theta, \frac{7\pi}{8}, \frac{9\pi}{8}\}, \text{Frame} \to \text{True}, \\
\quad \text{FrameLabel} \to \{\text{"Radians"}, \text{"Velocity (m/s)"}\}, \text{PlotLabel} \to \text{"Region C"}, \text{LabelStyle} \to \text{Magenta}] \\
\text{AccelerationC} & := r''[\theta] - r'[\theta] \times (\theta')^2 \\
\text{a}_1(\theta) & := r''[\theta] \\
\text{a}_2(\theta) & := r(\theta) \times (\theta'') + 2 \times r'[\theta] \times (\theta') \\
\text{a}_3(\theta) & := 974.53 \\
\text{a}_4(\theta) & := 0.
\end{align*}
\[ a_x = a_0 \cos(\theta) + a_n \cos(\theta + \frac{\pi}{2}) \]
\[ a_y = a_0 \sin(\theta) + a_n \sin(\theta + \frac{\pi}{2}) \]

\[ F_{x3} = m \, a_x \]
\[ F_{y3} = m \, a_y \]

0. \(-974.53 \cos(\theta)\)
0. \(-974.53 \sin(\theta)\)
0.1 \((0. - 974.53 \cos(\theta))\)
0.1 \((0. - 974.53 \sin(\theta))\)

\[ \text{AccelerationC} = \text{Plot}\left[a_x, a_y, \{\theta, \frac{7\pi}{8}, \frac{9\pi}{8}\}\right], \ Frame \rightarrow \text{True}, \]
FrameLabel -> {"Radians", "Acceleration (m/s²)"}, PlotLabel -> "Region C", LabelStyle -> Magenta]

\[ \text{ForceC} = \text{Plot}\left[F_{x3}, F_{y3}, \{\theta, \frac{7\pi}{8}, \frac{9\pi}{8}\}\right], \ Frame \rightarrow \text{True}, \]
FrameLabel -> {"Radians", "Force (N)"}, PlotLabel -> "Region C", LabelStyle -> Magenta]

Region D \((\alpha_1 + \beta_1 + \alpha_2 < \theta < \alpha_1 + \beta_1 + \alpha_2 + \beta_2)\)

\[ r(\theta) := (\theta \geq \alpha_1 + \beta_1 + \alpha_2) \& \& (\theta < \alpha_1 + \beta_1 + \alpha_2 + \beta_2) \]
\[ r(\theta) := (1/2)(r_2 - r_1) \left( 1 + \cos \left[ \frac{\theta - (\alpha_1 + \beta_1 + \alpha_2)}{\beta_2} \right] \right) + r_1 \]
\[ r(\theta) \]
\[ 0.0762 + 0.00635 \left( 1 + \cos \left( \frac{9 \pi}{8} + \theta \right) \right) \]

\[ r'[\theta] = -0.0508 \sin \left( \frac{9 \pi}{8} + \theta \right) \]

\[ r''[\theta] = -0.0464 \cos \left( \frac{9 \pi}{8} + \theta \right) \]

\[ \text{ProfileD} = \text{Plot} \left[ r[\theta], \left\{ \theta, \frac{9 \pi}{8}, \frac{10 \pi}{8} \right\}, \text{Frame} \to \text{True}, \right. \]

\[ \left. \text{FrameLabel} \to \{ "Radians", "Radius (m)" \}, \text{PlotLabel} \to "\text{Region D}" \right\}, \text{LabelStyle} \to \text{Blue} \]

\[ v_r[\theta] := r'[\theta] \]

\[ v_r[\theta] \]

\[ v_r[\theta] := r[\theta] \times (\theta') \]

\[ v_r[\theta] \]

\[ v_r = v_r[\theta] \cos[\theta] + v_\theta[\theta] \cos[\theta + \frac{\pi}{2}] \]

\[ v_\theta = v_r[\theta] \sin[\theta] + v_\theta[\theta] \sin[\theta + \frac{\pi}{2}] \]

\[ -0.0508 \sin \left( \frac{9 \pi}{8} + \theta \right) \]

\[ 104.7 \left( 0.0762 + 0.00635 \left( 1 + \cos \left( \frac{9 \pi}{8} + \theta \right) \right) \right) \]

\[ -104.7 \left( 0.0762 + 0.00635 \left( 1 + \cos \left( \frac{9 \pi}{8} + \theta \right) \right) \right) \sin[\theta] - 0.0508 \cos[\theta] \sin \left( \frac{9 \pi}{8} + \theta \right) \]

\[ 104.7 \cos[\theta] \left( 0.0762 + 0.00635 \left( 1 + \cos \left( \frac{9 \pi}{8} + \theta \right) \right) \right) - 0.0508 \sin[\theta] \sin \left( \frac{9 \pi}{8} + \theta \right) \]

\[ \text{VelocityD} = \text{Plot} \left[ [v_r, v_\theta], \left\{ \theta, \frac{9 \pi}{8}, \frac{10 \pi}{8} \right\}, \text{Frame} \to \text{True}, \right. \]

\[ \left. \text{FrameLabel} \to \{ "Radians", "Velocity (m/s)" \}, \text{PlotLabel} \to "\text{Region D}" \right\}, \text{LabelStyle} \to \text{Blue} \]
\[ a_\theta = r^2 \cdot [\theta] - r \cdot \cos(\theta) \cdot (\theta')^2 \]

\[ a_\theta = r \cdot \sin(\theta) + 2 \cdot r' \cdot \cos(\theta) \cdot (\theta') \]

\[ a_\theta = -0.4064 \cos\left(8\left[\frac{9\pi}{8} + \theta\right] - 10.9621 \left(0.0762 + 0.00635 \left[1 + \cos\left(8\left[\frac{9\pi}{8} + \theta\right]\right]\right]\right)\]

\[ -10.6375 \sin\left(8\left[\frac{9\pi}{8} + \theta\right]\right) \]

\[ a_x = a_\theta \cdot \cos(\theta) + a_\theta \cdot \cos\left(\frac{\pi}{2}\right) \]

\[ a_y = a_\theta \cdot \sin(\theta) + a_\theta \cdot \sin\left(\frac{\pi}{2}\right) \]

\[ F_{x,1} = m \cdot a_x \]

\[ F_{y,1} = m \cdot a_y \]

\[ \text{Cos}(\theta) \left(-0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10.9621 \left(0.0762 + 0.00635 \left[1 + \cos\left(\frac{9\pi}{8} + \theta\right]\right]\right]\right) + 10.6375 \sin(\theta) \cdot \sin\left[\frac{9\pi}{8} + \theta\right] \]

\[ \left(-0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10.9621 \left(0.0762 + 0.00635 \left[1 + \cos\left(\frac{9\pi}{8} + \theta\right]\right]\right]\right) \sin(\theta) - 10.6375 \cos(\theta) \cdot \sin\left[\frac{9\pi}{8} + \theta\right] \]

\[ 0.1 \left(\cos(\theta) \left(-0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10.9621 \left(0.0762 + 0.00635 \left[1 + \cos\left(\frac{9\pi}{8} + \theta\right]\right]\right]\right) \right) + 10.6375 \sin(\theta) \cdot \sin\left[\frac{9\pi}{8} + \theta\right] \]

\[ 0.1 \left(-0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10.9621 \left(0.0762 + 0.00635 \left[1 + \cos\left(\frac{9\pi}{8} + \theta\right]\right]\right]\right) \sin(\theta) - 10.6375 \cos(\theta) \cdot \sin\left[\frac{9\pi}{8} + \theta\right] \]

\[ \text{AccelerationD} = \text{Plot}\left(\left[a_x, a_y\right], \left(\theta, \frac{9\pi}{8}, \frac{10\pi}{8}\right), \text{Frame} \rightarrow \text{True}\right) \]

\[ \text{FrameLabel} \rightarrow \{"Radians", "Acceleration (m/s^2)"\}, \text{PlotLabel} \rightarrow \"Region D\", \text{LabelStyle} \rightarrow \text{Blue}\]
Region D

\[
\text{ForceD} = \text{Plot}\left[\{F_{x, D}, F_{y, D}\}, \left\{\theta, \frac{9\pi}{8}, \frac{10\pi}{8}\right\}, \text{Frame} \to \text{True}, \right.

\text{FrameLabel} \to \{"\text{Radians}"}, \text{"\text{Force (N)}"}, \text{PlotLabel} \to \"\text{Region D}\", \text{LabelStyle} \to \text{Blue}\left\}
\]

Region E (\(\alpha_1 + \beta_1 + \alpha_2 + \beta_2 < \theta < 2\pi\))

\[
\begin{align*}
\tau[\theta, ] &= (\theta = \alpha_1 + \beta_1 + \alpha_2 + \beta_2 \land \theta < 2\pi) \\
r[\theta, ] &= r_1 \\
r''[\theta] \\
\tau''[\theta]
\end{align*}
\]

ProfileE = Plot\left[\{r[\theta]\}, \left\{\theta, \frac{10\pi}{8}, \frac{16\pi}{8}\right\}, \text{Frame} \to \text{True}, \right.

\text{FrameLabel} \to \{"\text{Radians}"}, \text{"\text{Radius (m)}"}, \text{PlotLabel} \to \"\text{Region E}\", \text{LabelStyle} \to \text{Green}\left\}
\]
\[ v_r[\theta] := r'[\theta] \]
\[ v_\theta[\theta] \]
\[ v_r[\theta] := r[\theta] \times (\theta') \]
\[ v_\theta[\theta] \]
\[ v_x = v_r[\theta] \cos[\theta] + v_\theta[\theta] \cos[\theta + \frac{\pi}{2}] \]
\[ v_y = v_r[\theta] \sin[\theta] + v_\theta[\theta] \sin[\theta + \frac{\pi}{2}] \]

\[ 0 \]
\[ 7.97814 \]
\[ -7.97814 \sin[\theta] \]
\[ 7.97814 \cos[\theta] \]

VelocityE = Plot[\{v_x, v_y\}, \{\theta, 0, \frac{10\pi}{8}, \frac{16\pi}{8}\}, Frame -> True, FrameLabel -> \{"Radians", "Velocity (m/s)"\}, PlotLabel -> "Region E", LabelStyle -> Green \]

\[ a_r[\theta] := r''[\theta] = r[\theta] \times (\theta'') + 2 \]
\[ a_\theta[\theta] \]
\[ a_r[\theta] := r[\theta] \times (\theta'') + 2 \times r'[\theta] \times (\theta') \]
\[ a_\theta[\theta] \]

\[ -835.311 \]
\[ 0. \]
\[ a_x = a_x(\theta) \cos(\theta) + a_y(\theta) \cos\left(\theta + \frac{\pi}{2}\right) \]
\[ a_y = a_x(\theta) \sin(\theta) + a_y(\theta) \sin\left(\theta + \frac{\pi}{2}\right) \]

\[ F_{ax} = m \cdot a_x \]
\[ F_{ay} = m \cdot a_y \]

0. \cdot 835.311 \cos(\theta)
0. \cdot 835.311 \sin(\theta)
0.1 (0. \cdot 835.311 \cos(\theta))
0.1 (0. \cdot 835.311 \sin(\theta))

\[
\frac{625 \cos(\theta)}{g} \quad \frac{625 \cos(\theta)}{g}
\]

AccelerationE = Plot[\[\{a_x, a_y\}, \{\theta, 0, \frac{10\pi}{8}, \frac{16\pi}{8}\}\], Frame -> True,
FrameLabel -> {"Radiants", "Acceleration (m/s^2)"}, PlotLabel -> "Region E", LabelStyle -> Green]

ForceE = Plot[\[\{F_{ax}, F_{ay}\}, \{\theta, 0, \frac{10\pi}{8}, \frac{16\pi}{8}\}\], Frame -> True,
FrameLabel -> {"Radiants", "Force (N)"}, PlotLabel -> "Region E", LabelStyle -> Green]
ProfileAll = Show[ProfileA, ProfileB, ProfileC, ProfileD, ProfileE,
PlotRange -> Automatic, Frame -> True, FrameLabel -> {"Radiants", "Radius (m)"},
PlotLabel -> "Profile", LabelStyle -> Black, GridLines -> {\{(0, Dashed), \{(3\pi / 4, Dashed), \{(6\pi / 8, Dashed), \{(7\pi / 8, Dashed), \{(9\pi / 8, Dashed), \{(10\pi / 8, Dashed), \{(16\pi / 8, Dashed}, \{(1, Orange), \{(1, Orange)\}\}\]\]} \]

VelocityAll = Show[VelocityA, VelocityB, VelocityC, VelocityD, VelocityE,
PlotRange -> Automatic, Frame -> True, FrameLabel -> {"Radiants", "Velocity (m/s)"},
PlotLabel -> "Velocity", LabelStyle -> Black, GridLines -> {\{(0, Dashed), \{(3\pi / 4, Dashed), \{(6\pi / 8, Dashed), \{(7\pi / 8, Dashed), \{(9\pi / 8, Dashed), \{(10\pi / 8, Dashed), \{(16\pi / 8, Dashed}, \{(-8, Dashed), \{(8, Dashed)\}\]\]} \]
AccelerationAll = Show[AccelerationA, AccelerationB, AccelerationC, AccelerationD, 
  AccelerationE, PlotRange -> Automatic, Frame -> True, FrameLabel -> {"Radians", "Acceleration (m/s^2)"}, 
  PlotLabel -> "Acceleration", LabelStyle -> Black, GridLines -> 
  ({0, Dashed}, {3 \[Pi]/4, Dashed}, {6 \[Pi]/8, Dashed}, 
  {7 \[Pi]/8, Dashed}, {9 \[Pi]/8, Dashed}, {10 \[Pi]/8, Dashed}, 
  {16 \[Pi]/8, Dashed}, {{-850, Dashed}, {850, Dashed}}); 
  Acceleration
\text{ForceAll} = \text{Show}[\text{ForceA, ForceB, ForceC, ForceD, ForceE,}
\]
\text{PlotRange \rightarrow Automatic, Frame \rightarrow True, FrameLabel \rightarrow \{"Radians", "Force (N)"},
\]
\text{PlotLabel \rightarrow "Force", LabelStyle \rightarrow Black, GridLines \rightarrow \{(0, Dashed), \{\frac{\pi}{4}, Dashed\}, \{\frac{6\pi}{8}, Dashed\},}
\]
\{\frac{7\pi}{8}, Dashed\}, \{\frac{9\pi}{8}, Dashed\}, \{\frac{10\pi}{8}, Dashed\}, \{\frac{16\pi}{8}, Dashed\}, \{\{\text{85, Dashed}, \{\text{85, Dashed}}\}\}
\]

\begin{align*}
\text{Shifted By 180 degrees Region A (0} &< \theta < \alpha_1) \\
\text{r}[\theta] := (\theta &\geq 0 \&\& \theta < \alpha_1) - \pi \\
r[\theta] := r_2 \\
r^*[\theta] \\
r^{**}[\theta] \\
0 \\
0
\end{align*}

\text{ProfileAPI} = \text{Plot}[\{r[\theta]\}, \{0, 0, \frac{\pi}{8}\}, \text{Frame \rightarrow True, FrameLabel \rightarrow \{"Radians", "Radius (m)"}},
\]
\text{PlotLabel \rightarrow "Shifted By 180 degrees Region A", LabelStyle \rightarrow Orange} \]
\( v_r[\theta] := r'[\theta] \)
\( v_\theta[\theta] \)
\( v_{\theta^2}[\theta] := r[\theta] * (\theta'') \)
\( v_{\theta^3}[\theta] \)

\[
v_r = -v_r[\theta] \cos[\theta] + v_{\theta^2}[\theta] \cos[\theta + \frac{\pi}{2}]
\]

\[
v_\theta = -v_r[\theta] \sin[\theta] + v_{\theta^2}[\theta] \sin[\theta + \frac{\pi}{2}]
\]

0
9.30783
9.30783 \sin[\theta]

\(-9.30783 \cos[\theta]\)

Velocity API := Plot[\{v_r, v_\theta\}, \{\theta, 0, \frac{\pi}{8}\}, Frame \rightarrow True,
FrameLabel \rightarrow \{"Radians", "Velocity (m/s)"\}, PlotLabel \rightarrow "Shifted By 180 degrees Region A", LabelStyle \rightarrow Orange] \]

\[
a_r[\theta] := r''[\theta] - r[\theta] * (\theta'')^2
\]
\( a_\theta[\theta] \)
\( a_{\theta^2}[\theta] := r[\theta] * (\theta'') + 2 * r'[\theta] * (\theta'') \)
\( a_{\theta^3}[\theta] \)

\(-974.53\)

0.
\[a_x = -\left(a_{x1}[0] \cos[\theta] + a_{x2}[0] \cos\left[\theta + \frac{\pi}{2}\right]\right)\]
\[a_y = -\left(a_{y1}[0] \sin[\theta] + a_{y2}[0] \sin\left[\theta + \frac{\pi}{2}\right]\right)\]

\[F_{xx} = m \cdot a_x\]
\[F_{yy} = m \cdot a_y\]

\[0.1(0. + 974.53 \cos[\theta])\]
\[0.1(0. + 974.53 \sin[\theta])\]

\[\text{AccelerationAPI} = \text{Plot}\left(a_x, a_y, \{\theta, 0, \frac{\pi}{8}\}\right), \text{Frame} \to \text{True},\]
\[\text{FrameLabel} \to \{"Radians", "Acceleration (m/s^2)"}, \text{PlotLabel} \to \"Shifted By 180 degrees Region A", \text{LabelStyle} \to \text{Orange}\]

\[\text{ForceAPI} = \text{Plot}\left(F_{xx}, F_{yy}, \{\theta, 0, \frac{\pi}{8}\}\right), \text{Frame} \to \text{True},\]
\[\text{FrameLabel} \to \{"Radians", "Force (N)"}, \text{PlotLabel} \to \"Region A", \text{LabelStyle} \to \text{Orange}\]

\[0.1(0. + 835.311 \cos[\theta]) + 0.1(0. + 974.53 \cos[\theta])\]
0.1 (0. – 835.311 \sin(\theta)) + 0.1 (0. + 974.53 \sin(\theta))

![Graph of region A](image)

Shifting By 180 degrees Region B \((\alpha_1 < \theta < a_1 + \beta_1)\)

\[
r[\theta] := \begin{cases} 
  \theta > \alpha_1 & \& \theta < \alpha_1 + \beta_1 - \pi \\
  (1/2)(r_2 - r_1) \left[ 1 - \cos(\pi(\theta - a_1) / \beta_1) \right] + r_1
\end{cases}
\]

\[
r[\theta] = 0.0762 + 0.00635 \left[ 1 - \cos \left( \frac{3\pi}{4} + \theta \right) \right]
\]

\[
r'[\theta] = 0.0508 \sin \left( \frac{3\pi}{4} + \theta \right)
\]

\[
r''[\theta] = 0.4064 \cos \left( \frac{3\pi}{4} + \theta \right)
\]

ProfileBP1 = Plot\[r[\theta], \{\theta, 0, \frac{\pi}{8}, \frac{2\pi}{8}\}, \text{Frame} \to \text{True}, \text{FrameLabel} \to \{"Radius\"}, \text{PlotLabel} \to \text{"Shifted By 180 degrees Region B"}, \text{LabelStyle} \to \text{Red}\]

![Graph of region B](image)
\[ v_\theta[\theta] := r'[\theta] \]
\[ v_r[\theta] := r[\theta] \]
\[ v_\phi[\theta] := r[\theta] \ast (\theta') \]
\[ v_\phi[\theta] = \left( v_\phi[\theta] \ast \cos[\theta] + v_\phi[\theta] \ast \cos[\theta + \frac{3\pi}{4}] \right) \]
\[ v_\theta = \left( v_\phi[\theta] \ast \sin[\theta] + v_\phi[\theta] \ast \sin[\theta + \frac{3\pi}{4}] \right) \]
\[ 0.0508 \sin\left( \frac{3\pi}{4} + \theta \right) \]
\[ 104.7 \left( 0.0762 + 0.00635 \left( 1 - \cos\left( \frac{3\pi}{4} + \theta \right) \right) \right) \]
\[ 104.7 \left( 0.0762 + 0.00635 \left( 1 - \cos\left( \frac{3\pi}{4} + \theta \right) \right) \right) \sin[\theta] - 0.0508 \cos[\theta] \sin\left[ \frac{3\pi}{4} + \theta \right] \]
\[ -104.7 \cos[\theta] \left( 0.0762 + 0.00635 \left( 1 - \cos\left( \frac{3\pi}{4} + \theta \right) \right) \right) - 0.0508 \sin[\theta] \sin\left[ \frac{3\pi}{4} + \theta \right] \]

VelocityBPi := Plot \{ v_\theta, v_\phi \}, \{ \theta, \frac{\pi}{8}, \frac{3\pi}{8} \} \}

FrameLabel \rightarrow \{ "Radians", "Velocity (m/s)" \}, PlotLabel \rightarrow \"Shifted By 180 degrees Region B\", LabelStyle \rightarrow \ nowrap \}

\[ a_\phi[\theta] := r''[\theta] - r[\theta] \ast (\theta')^2 \]
\[ a_\theta[\theta] := r\theta'[\theta] + 2 \ast r[\theta] \ast (\theta') \]
\[ a_\phi[\theta] = \left( a_\phi[\theta] \ast \cos[\theta] + a_\phi[\theta] \ast \cos[\theta + \frac{3\pi}{4}] \right) \]
\[ a_\theta = \left( a_\phi[\theta] \ast \sin[\theta] + a_\phi[\theta] \ast \sin[\theta + \frac{3\pi}{4}] \right) \]
\[ F_x = m \cdot a_\phi \]
\[ F_y = m \cdot a_\theta \]
\[ -\cos[\theta] \left( 104.7 \left( 0.0762 + 0.00635 \left( 1 - \cos\left( \frac{3\pi}{4} + \theta \right) \right) \right) + 0.4064 \cos\left( \frac{3\pi}{4} + \theta \right) \right) + 10.6375 \sin[\theta] \sin\left[ \frac{3\pi}{4} + \theta \right] \]
\[-10.962.1 \left( 0.0762 + 0.00635 \left( 1 - \cos \left( \frac{3\pi}{4} + \theta \right) \right) \right) + 0.4064 \cos \left( \frac{3\pi}{4} + \theta \right) + 10.6375 \sin[\theta] x 0.1 \left( -\cos[\theta] -10.962.1 \left( 0.0762 + 0.00635 \left( 1 - \cos \left( \frac{3\pi}{4} + \theta \right) \right) \right) + 0.4064 \cos \left( \frac{3\pi}{4} + \theta \right) + 10.6375 \sin[\theta] x 0.1 \left( -\cos[\theta] -10.962.1 \left( 0.0762 + 0.00635 \left( 1 - \cos \left( \frac{3\pi}{4} + \theta \right) \right) \right) + 0.4064 \cos \left( \frac{3\pi}{4} + \theta \right) + 10.6375 \sin[\theta] \right)\]
Shifted By 180 degrees Region B

Shifted By 180 degrees Region C \((\alpha_1 + \beta_1 < \theta < \alpha_1 + \beta_1 + \alpha_2)\)

\[
r[\theta] := (\theta > \alpha_1 + \beta_1 \&\& \theta < \alpha_1 + \beta_1 + \alpha_2) - \pi
\]

\[
r[\theta] := r_1
\]

\[
r''[\theta] = \text{ProfileCPI} = \text{Plot}\left[|r[\theta]|, \left\{\theta, \frac{2\pi}{8}, \frac{14\pi}{8}\right\}, \text{Frame} \rightarrow \text{True}, \text{FrameLabel} \rightarrow \{"Radians", "Radius (m)"}, \text{PlotLabel} \rightarrow \text{"Shifted By 180 degrees Region C"}, \text{LabelStyle} \rightarrow \text{Magenta}\right]
\]

0

Shifted By 180 degrees Region C

\[
v_x[\theta] := r'[\theta] = 0
\]

\[
v_y[\theta] := r[\theta] \times (\theta') = 0
\]

\[
v_x = -\left(v_x[\theta] \cos[\theta] + v_y[\theta] \cos[\theta + \frac{\pi}{2}]\right)
\]

\[
v_y = -\left(v_x[\theta] \sin[\theta] + v_y[\theta] \sin[\theta + \frac{\pi}{2}]\right)
\]

0

7.97814
\[ 7.97814 \sin[\theta] \]
\[-7.97814 \cos[\theta] \]

Velocity CP1 \(=\) Plot \([f_x, f_y], \{\theta, \frac{2\pi}{8}, \frac{14\pi}{8}\}, \text{Frame} \rightarrow \text{True}, \)
\[\text{FrameLabel} \rightarrow \{"Radians", "Velocity (m/s)"\}, \text{PlotLabel} \rightarrow \"Shifted By 180 degrees Region C\", \text{LabelStyle} \rightarrow \text{Magenta}\]

\[ a_{x}[\theta] := r \cdot v[\theta] - r[\theta] \cdot (\theta')^2 / 2 \]
\[ a_{y}[\theta] := r[\theta] \cdot (\theta'') + 2 \cdot r'[\theta] \cdot (\theta') \]
\[ a_{z}[\theta] := -835.311 \]
\[ 0. \]
\[ a_{x} := \left( a_{x}[\theta] \cos[\theta] + a_{y}[\theta] \cos[\theta + \frac{\pi}{2}] \right) \]
\[ a_{y} := \left( a_{x}[\theta] \sin[\theta] + a_{y}[\theta] \sin[\theta + \frac{\pi}{2}] \right) \]
\[ F_{xx} = m \cdot a_{x} \]
\[ F_{yy} = m \cdot a_{y} \]
\[ 0. + 835.311 \cos[\theta] \]
\[ 0. + 835.311 \sin[\theta] \]
\[ 0.1 (0. + 835.311 \cos[\theta]) \]
\[ 0.1 (0. + 835.311 \sin[\theta]) \]

Acceleration CP1 \(=\) Plot \([a_x, a_y], \{\theta, \frac{2\pi}{8}, \frac{14\pi}{8}\}, \text{Frame} \rightarrow \text{True}, \)
\[\text{FrameLabel} \rightarrow \{"Radians", "Acceleration (m/s^2)"\}, \text{PlotLabel} \rightarrow \"Shifted By 180 degrees Region C\", \text{LabelStyle} \rightarrow \text{Magenta}\]
\begin{align*}
\text{ForceCPI} &= \text{Plot}\left( [F_x, F_y], \{\theta, \frac{2\pi}{8}, \frac{14\pi}{8}\}, \text{Frame} \to \text{True}, \ \\
\text{FrameLabel} &\to \{ \text{"Radians"}, \text{"Force (N)"} \}, \text{PlotLabel} \to \text{"Shifted By 180 degrees Region C"}, \text{LabelStyle} \to \text{Magenta} \right) \\
\text{ForceCd1} &= \text{Plot}\left( [F_x, F_y], \{\theta, \frac{2\pi}{8}, \frac{6\pi}{8}\}, \text{Frame} \to \text{True}, \ \\
\text{FrameLabel} &\to \{ \text{"Radians"}, \text{"Force (N)"} \}, \text{PlotLabel} \to \text{"Shifted By 180 degrees Region C"}, \text{LabelStyle} \to \text{Magenta} \right) \\

F_{xd} &= F_x + F_{x1} \\
F_{yd} &= F_y + F_{y1} \\
\text{ForceCd1} &= \text{Plot}\left( [F_x, F_y], \{\theta, \frac{2\pi}{8}, \frac{6\pi}{8}\}, \text{Frame} \to \text{True}, \ \\
\text{FrameLabel} &\to \{ \text{"Radians"}, \text{"Force (N)"} \}, \text{PlotLabel} \to \text{"Shifted By 180 degrees Region C"}, \text{LabelStyle} \to \text{Magenta} \right) \\
0.1 \cdot (0. - 835.311 \cos(\theta)) + 0.1 \cdot (0. + 835.311 \cos(\theta)) \\
0.1 \cdot (0. - 835.311 \sin(\theta)) + 0.1 \cdot (0. + 835.311 \sin(\theta))
\end{align*}
\[ F_{\text{ad}} = F_{v5} + F_{v2} \]
\[ F_{\text{vd}} = F_{v5} + F_{v2} \]
\[ \text{Frame} \to \text{True} \]
\[ \text{FrameLabel} \to \{ "\text{"Radians"}, "\text{"Force (N)"}"}, \text{PlotLabel} \to "\text{Shifted By 180 degrees Region C}" , \text{LabelStyle} \to \text{Magenta} \]

\[
0.1 \left( (0. + 835.311 \cos(\theta)) + 
0.1 \left( \cos(\theta) \left(-10.962.1 \left(0.0762 + 0.00635 \left(1 - \cos\left(\frac{3 \pi}{4} + \theta\right)\right)\right) + 0.4064 \cos\left(8\left(\frac{3 \pi}{4} + \theta\right)\right)\right) - 10.6375 \sin(\theta) \sin\left(\frac{3 \pi}{4} + \theta\right)\right)\right)
\]

\[
0.1 \left( (0. + 835.311 \sin(\theta)) + 
0.1 \left( \left(-10.962.1 \left(0.0762 + 0.00635 \left(1 - \cos\left(\frac{3 \pi}{4} + \theta\right)\right)\right) + 0.4064 \cos\left(8\left(\frac{3 \pi}{4} + \theta\right)\right)\right) \sin(\theta) + 10.6375 \cos(\theta) \sin\left(\frac{3 \pi}{4} + \theta\right)\right)\right)
\]

\[ F_{\text{ad}} = F_{v5} + F_{v3} \]
\[ F_{\text{vd}} = F_{v5} + F_{v3} \]
\[ \text{Frame} \to \text{True} \]
\[ \text{FrameLabel} \to \{ "\text{"Radians"}, "\text{"Force (N)"}"}, \text{PlotLabel} \to "\text{Shifted By 180 degrees Region C}" , \text{LabelStyle} \to \text{Magenta} \]

\[
0.1 \left( (0. - 974.53 \cos(\theta)) + 0.1 \left( 0. + 835.311 \cos(\theta)\right)\right)
\]
\[
0.1 \left( (0. - 974.53 \sin(\theta)) + 0.1 \left( 0. + 835.311 \sin(\theta)\right)\right)
\]
\[ F_{sd} = F_{s5} + F_{s6} \]
\[ F_{sd} = F_{s1} + F_{s6} \]

\[ \text{ForceCd4} = \text{Plot}\left[F(x), \{x, 9\pi/8, 10\pi/8\}, \text{Frame} \rightarrow \text{True}, \right. \]
\[ \left. \text{FrameLabel} \rightarrow \{"\text{Radians}\}, \"\text{Force (N)}\"\}, \text{PlotLabel} \rightarrow \"\text{Shifted By 180 degrees Region C\}, \text{LabelStyle} \rightarrow \text{Magenta}\right] \]

\[ 0.1 (0. + 835.311 \cos[\theta]) + 0.1 \left(\cos[\theta] - 0.4064 \cos\left(\frac{9\pi}{8} + \phi\right) - 10.9621 \left(0.0762 + 0.00635 \left(1 + \cos\left(\frac{9\pi}{8} + \phi\right)\right)\right) + 10.6375 \sin[\theta] \sin\left(\frac{9\pi}{8} + \phi\right)\right) \]

\[ 0.1 (0. + 835.311 \sin[\theta]) + 0.1 \left(-0.4064 \cos\left(\frac{9\pi}{8} + \phi\right) - 10.9621 \left(0.0762 + 0.00635 \left(1 + \cos\left(\frac{9\pi}{8} + \phi\right)\right)\right) \sin[\theta] - 10.6375 \cos[\theta] \sin\left(\frac{9\pi}{8} + \phi\right)\right) \]

\[ F_{sd} = F_{s5} + F_{s6} \]
\[ F_{sd} = F_{s1} + F_{s6} \]

\[ \text{ForceCd5} = \text{Plot}\left[F(x), \{x, 10\pi/8, 14\pi/8\}, \text{Frame} \rightarrow \text{True}, \right. \]
\[ \left. \text{FrameLabel} \rightarrow \{"\text{Radians}\}, \"\text{Force (N)}\"\}, \text{PlotLabel} \rightarrow \"\text{Shifted By 180 degrees Region C\}, \text{LabelStyle} \rightarrow \text{Magenta}\right] \]

\[ 0.1 (0. - 835.311 \cos[\theta]) + 0.1 (0. + 835.311 \cos[\theta]) \]

\[ 0.1 (0. - 835.311 \sin[\theta]) + 0.1 (0. + 835.311 \sin[\theta]) \]
Shifted By 180 degrees Region C

\[
\begin{align*}
\theta[\theta] &:= (\theta \geq \alpha_1 + \beta_1 + \alpha_2 \& \& \theta < \alpha_1 + \beta_1 + \alpha_2 + \beta_2 ) - \pi \\
r[\theta] &:= (1/2)(2 - r_1) \left( 1 + \cos \left( \frac{\pi}{\beta_1} \right) \right) + r_1 \\
r[\theta] &:= 0.0762 + 0.00635 \left( 1 + \cos \left( \frac{9\pi}{8} + \theta \right) \right) \\
r'[\theta] &:= -0.0508 \sin \left( \frac{9\pi}{8} + \theta \right) \\
r''[\theta] &:= -0.4064 \cos \left( \frac{9\pi}{8} + \theta \right)
\end{align*}
\]

ProfileDPI = Plot[\{r[\theta], \theta, \frac{14\pi}{8}, \frac{15\pi}{8} \}, Frame -> True,

FrameLabel -> \{"Radians", "Radius (m)"\}, PlotLabel -> "Shifted By 180 degrees Region D", LabelStyle -> Blue]
\[ v_r(\theta) := r'(\theta) \]
\[ v_{\theta}(\theta) := r(\theta) \cdot (\theta') \]
\[ v_\theta := \left( v_r(\theta) \cos(\theta) + v_{\theta}(\theta) \cos\left(\frac{\pi}{2} + \frac{\theta}{2}\right) \right) \]
\[ v_r := \left( v_r(\theta) \sin(\theta) + v_{\theta}(\theta) \sin\left(\frac{\pi}{2} + \frac{\theta}{2}\right) \right) \]

\[-0.0508 \sin\left(\frac{9\pi}{8} + \theta\right) \]

\[
\begin{align*}
104.7 \left(0.0762 + 0.00635 \left(1 + \cos\left(\frac{9\pi}{8} + \theta\right)\right)\right) \\
104.7 \left(0.0762 + 0.00635 \left(1 + \cos\left(\frac{9\pi}{8} + \theta\right)\right)\sin(\theta) + 0.0508 \cos(\theta) \sin\left(\frac{9\pi}{8} + \theta\right)\right) \\
-104.7 \cos(\theta) \left(0.0762 + 0.00635 \left(1 + \cos\left(\frac{9\pi}{8} + \theta\right)\right)\right) + 0.0508 \sin(\theta) \sin\left(\frac{9\pi}{8} + \theta\right) \\
\end{align*}
\]

Velocity DPI = Plot\left(\{v_r, v_\theta\}, \{\theta, \frac{14\pi}{8}, \frac{15\pi}{8}\}\right), Frame \to True, FrameLabel \to \{"Radians", "Velocity (m/s)"\}, PlotLabel \to \"Shifted By 180 degrees Region D\", LabelStyle \to Blue\]

\[ a_r(\theta) := r''(\theta) - r(\theta) \cdot (\theta')^2 \]
\[ a_{\theta}(\theta) := r(\theta) \cdot (\theta') + 2 \cdot r'(\theta) \cdot (\theta')' \]
\[ a_\theta := \left( a_r(\theta) \cos(\theta) + a_{\theta}(\theta) \cos\left(\frac{\pi}{2} + \frac{\theta}{2}\right) \right) \]
\[ a_r := \left( a_r(\theta) \sin(\theta) + a_{\theta}(\theta) \sin\left(\frac{\pi}{2} + \frac{\theta}{2}\right) \right) \]

\[ F_{nx} = m a_r \]
\[ F_{ny} = m a_\theta \]

\[-\cos(\theta) \left(-0.4064 \cos\left(\frac{9\pi}{8} + \theta\right) - 10.9621 \left(0.0762 + 0.00635 \left(1 + \cos\left(\frac{9\pi}{8} + \theta\right)\right)\right)\right) - 10.6375 \sin(\theta) \sin\left(\frac{9\pi}{8} + \theta\right) \]
\[-0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10962.1 \left\{0.0762 + 0.00635 \left[1 + \cos\left[\frac{9\pi}{8} + \theta\right]\right]\right\} \sin[\theta] + 10.6375 \cos[\theta] \sin\left[\frac{9\pi}{8} + \theta\right]\]

\[0.1 \left\{-\cos[\theta] - 0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10962.1 \left\{0.0762 + 0.00635 \left[1 + \cos\left[\frac{9\pi}{8} + \theta\right]\right]\right\} \sin[\theta] + 10.6375 \sin[\theta] \sin\left[\frac{9\pi}{8} + \theta\right]\right\}\]

\[0.1 \left\{-0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10962.1 \left\{0.0762 + 0.00635 \left[1 + \cos\left[\frac{9\pi}{8} + \theta\right]\right]\right\} \sin[\theta] + 10.6375 \cos[\theta] \sin\left[\frac{9\pi}{8} + \theta\right]\right\}\]

AccelerationDPI = Plot[{x[t, y[t]], {\theta, \frac{14\pi}{8}, \frac{15\pi}{8}}}, Frame -> True,
FrameLabel -> {"Radians", "Acceleration (m/s^2)"}, PlotLabel -> "Shifted By 180 degrees Region D", LabelStyle -> Blue]

ForceDPI = Plot[{F_x, F_y}, {\theta, \frac{14\pi}{8}, \frac{15\pi}{8}}, Frame -> True,
FrameLabel -> {"Radians", "Force (N)"}, PlotLabel -> "Shifted By 180 degrees Region D", LabelStyle -> Blue]

F_{ad} = F_x + F_y
F_{id} = F_x + F_y

ForceDd = Plot[{F_{ad}, F_{id}}, {\theta, \frac{14\pi}{8}, \frac{15\pi}{8}}, Frame -> True,
FrameLabel -> {"Radians", "Force (N)"}, PlotLabel -> "Shifted By 180 degrees Region D", LabelStyle -> Blue]

0.1 \left\{1 - 0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10962.1 \left\{0.0762 + 0.00635 \left[1 + \cos\left[\frac{9\pi}{8} + \theta\right]\right]\right\} \sin[\theta] + 10.6375 \sin[\theta] \sin\left[\frac{9\pi}{8} + \theta\right]\right\} +

0.1 \left\{-\cos[\theta] - 0.4064 \cos\left[\frac{9\pi}{8} + \theta\right] - 10962.1 \left\{0.0762 + 0.00635 \left[1 + \cos\left[\frac{9\pi}{8} + \theta\right]\right]\right\} \sin[\theta] + 10.6375 \sin[\theta] \sin\left[\frac{9\pi}{8} + \theta\right]\right\}\]
0.1 \cdot (0. - 835.311 \cos(\theta)) + \\
0.1 \cdot \left( -0.4064 \cos\left(\frac{9\pi}{8} + \theta\right) - 10.962.1 \left( 0.0762 + 0.00635 \left( 1 + \cos\left(\frac{9\pi}{8} + \theta\right) \right) \right) \sin(\theta) + 10.6375 \cos(\theta) \sin\left(\frac{9\pi}{8} + \theta\right) \right)

Shifted By 180 degrees Region E

\( r(\theta) := (\theta \geq \pi \land \theta \leq 2\pi) \)
\( r(\theta) := r_1 \)
\( r(\theta) := r_2 \)

ProfileEPi = Plot\{r(\theta), \{\theta, \frac{15\pi}{8}, \frac{16\pi}{8}\}\}, Frame -> True,
FrameLabel -> {"Radians", "Radius (m)"}, PlotLabel -> "Shifted By 180 degrees Region E", LabelStyle -> Green

0

0

Shifted By 180 degrees Region E

\( \nu_r(\theta) := r'(\theta) \)
\( \nu_r(\theta) := r'(\theta) \cdot \theta' \)
\( \nu_\theta(\theta) := \left( \nu_r(\theta) \cos(\theta) + \nu_\theta(\theta) \cos(\theta + \frac{\pi}{2}) \right) \)
\( \nu_\theta(\theta) := \left( \nu_r(\theta) \sin(\theta) + \nu_\theta(\theta) \sin(\theta + \frac{\pi}{2}) \right) \)

0

9.30783
Velocity EPI = Plot\left( \left\{ \frac{15\pi}{8}, \frac{16\pi}{8} \right\}, \text{Frame} \rightarrow \text{True}, \right.

FrameLabel \rightarrow \{ "Radians", "Velocity (m/s)" \}, PlotLabel \rightarrow "Shifted By 180 degrees Region E", LabelStyle \rightarrow \text{Green} \)

\begin{align*}
a_{\theta}[\theta] & := r^\prime[\theta] - r[\theta] \cdot (\theta^\prime)^2 \\
a_{\theta}[\theta] & := r[\theta] + (\theta^\prime)^2 + 2 \cdot r^\prime[\theta] \cdot (\theta^\prime) \\
a_{\theta}[\theta] & := -974.53 \\
0. & \\
a_\theta & = \left( a_{\theta}[\theta] \cos[\theta] + a_{\phi}[\theta] \cos\left[\theta + \frac{\pi}{2}\right] \right) \\
a_\phi & = \left( a_{\theta}[\theta] \sin[\theta] + a_{\phi}[\theta] \sin\left[\theta + \frac{\pi}{2}\right] \right) \\
F_{\text{ss}} & = m \cdot a_\theta \\
F_{\text{ss}} & = m \cdot a_\phi \\
0. & + 979.53 \cos[\theta] \\
0. & + 974.53 \sin[\theta] \\
0.1 \cdot (0. + 974.53 \cos[\theta]) & \\
0.1 \cdot (0. + 974.53 \sin[\theta]) & \\
\text{Acceleration EPI} = \text{Plot}\left[ \left\{ a_\theta, a_\phi \right\}, \left\{ 0, \frac{15\pi}{8}, \frac{16\pi}{8} \right\}, \text{Frame} \rightarrow \text{True}, \right.

FrameLabel \rightarrow \{ "Radians", "Acceleration (m/s^2)" \}, PlotLabel \rightarrow "Shifted By 180 degrees Region E", LabelStyle \rightarrow \text{Green} \)
\[
\text{ForceEPI} = \text{Plot}\left[\{F_{\theta\epsilon}, F_{\theta\varphi}\}, \left\{\theta, \frac{15\pi}{8}, \frac{16\pi}{8}\right\}, \text{Frame} \to \text{True}, \right.
\]
\[
\text{FrameLabel} \to \{"\text{Radians}"}, \"\text{Force (N)}\"), \text{PlotLabel} \to \"\text{Shifted By 180 degrees Region E}\", \text{LabelStyle} \to \text{Green}\]
\]

\[
\text{F_{ad}} = F_{\theta\epsilon} + F_{\varphi\epsilon}
\]
\[
F_{ad} = F_{\theta\varphi} + F_{\varphi\varphi}
\]
\[
\text{ForceEd} = \text{Plot}\left[\{F_{\theta\epsilon}, F_{\theta\varphi}\}, \left\{\theta, \frac{15\pi}{8}, \frac{16\pi}{8}\right\}, \text{Frame} \to \text{True}, \right.
\]
\[
\text{FrameLabel} \to \{"\text{Radians}"}, \"\text{Force (N)}\"), \text{PlotLabel} \to \"\text{Shifted By 180 degrees Region E}\", \text{LabelStyle} \to \text{Green}\]
\]

0.1(0. - 835.311 \cos(\theta) + 0.1(0. + 974.53 \cos(\theta))
\]
\[
0.1(0. - 835.311 \sin(\theta)) + 0.1(0. + 974.53 \sin(\theta))
\]
ProfileAllPl = Show[ProfileAPI, ProfileBPI, ProfileCPI, ProfileDPI, ProfileEPI, PlotRange → Automatic,
Frame → True, FrameLabel → {"Radii", "Radius (m)"}, PlotLabel → "Shifted By 180 degrees Profile",
LabelStyle → Black, GridLines → {{0, Dashed}, {π/8, Dashed}, {2π/8, Dashed},
{8π/8, Dashed}, {14π/8, Dashed}, {15π/8, Dashed}, {16π/8, Dashed}, 
{{1, Orange}, {1, Orange}]]

VelocityAllPl = Show[VelocityAPI, VelocityBPI, VelocityCPI, VelocityDPI, VelocityEPI, PlotRange → Automatic,
Frame → True, FrameLabel → {"Radii", "Velocity (m/s)"}, PlotLabel → "Shifted By 180 degrees Velocity",
LabelStyle → Black, GridLines → {{0, Dashed}, {π/8, Dashed}, {2π/8, Dashed}, {8π/8, Dashed},
{14π/8, Dashed}, {15π/8, Dashed}, {16π/8, Dashed}, {{1, 9.5}, Dashed}, {8, Dashed}]]
AccelerationAPI =

Show[AccelerationAPI, AccelerationBPI, AccelerationCPPI, AccelerationDPPI, AccelerationEPI, PlotRange → Automatic, Frame → True, FrameLabel → {"Radians", "Acceleration (m/s^2)"}, PlotLabel → "Shifted By 180 degrees Acceleration", LabelStyle → Black, GridLines → {({0, Dashed}, {\[Pi]/8, Dashed}, {\[Pi]/4, Dashed}, {3\[Pi]/8, Dashed}, {\[3\pi]/4, Dashed}, {15\[pi]/8, Dashed}, {3\[pi]/2, Dashed}, {16\[pi]/8, Dashed}), ({-800, Dashed}, {800, Dashed}, {950, Dashed})}]

Shifted By 180 degrees Acceleration
\textbf{Two Opposite Balls Force}

\begin{align*}
\text{Force} & = \text{Show} \left[ \text{ForceAd, ForceBd, ForceCd1, ForceCd2, ForceCd3, ForceCd4, ForceCd5, ForceCd6, PlotRange} \rightarrow \text{Automatic}, \right. \\
& \left. \text{Frame} \rightarrow \text{True}, \text{FrameLabel} \rightarrow \{ \text{"Radians", "Force (N)"} \}, \text{PlotLabel} \rightarrow \text{"Two Opposite Balls Force"}, \text{LabelStyle} \rightarrow \text{Black}, \right. \\
& \text{GridLines} \rightarrow \left\{ \{0, \text{Dashed}\}, \left\{ \frac{\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{2\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{3\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{6\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{7\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{9\pi}{8}, \text{Dashed}\right\}, \right. \\
& \left. \left\{ \frac{10\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{14\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{15\pi}{8}, \text{Dashed}\right\}, \left\{ \frac{16\pi}{8}, \text{Dashed}\right\}, \{[-7, \text{Dashed}], [14, \text{Dashed}]\} \right\} \}
\end{align*}
BIBLIOGRAPHY
