DYNAMIC EFFECTS OF PROPELLANT SLOSH ON A PLANETARY LANDER
WITH TRANSIENT MASS PROPERTIES

A Thesis

Presented to the faculty of the Department of Mechanical Engineering
California State University, Sacramento

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in

Mechanical Engineering

by

Thomas William Carlson

FALL
2012
DYNAMIC EFFECTS OF PROPELLANT SLOSH ON A PLANETARY LANDER
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Department of Mechanical Engineering
Abstract

of

DYNAMIC EFFECTS OF PROPELLANT SLOSH ON A PLANETARY LANDER

WITH TRANSIENT MASS PROPERTIES

by

Thomas William Carlson

NASA is currently in development of a vertical test bed vehicle demonstrating new green propellant propulsion systems and autonomous landing and hazard detection technology. The vehicle is required to land on the Earth’s surface without the aid of parachutes using a single gimbaled LOX/Methane engine to control its descent. One of the many challenges of this system is the ability to control the descent while the vehicle is exposed to external and internal disturbances, and under changing mass properties (propellant consumption). In order to develop a suitable control system, the maximum expected magnitude of the disturbance inputs must be predicted. One such disturbance is the propellant slosh created during vehicle accelerations. This paper analyzes the effects of propellant slosh on the rigid body motion of the vehicle for both liquid methane (fuel) and liquid oxygen (oxidizer) residing in spherical propellant tanks containing cruciform and rigid ring baffles. This was accomplished by analysis and software simulation. The
results are compared with the NASA models as well as existing historical data. With a validated analysis model, the software can be used to run multiple scenarios to gain information on vehicle response prior to testing using expensive vehicle hardware.

The software used for the simulation is SimWise 4D by Design Simulation Technologies, Inc. Modeling of the rigid body vehicle was done using Pro ENGINEER and Siemens NX8 CAD software.

Simulation in SimWise 4D showed that the induced vehicle rotation rates due to propellant slosh in the spherical tanks are dependent on the propellant fuel height, slosh frequency, slosh amplitude, and propellant mass. Induced angular rates and accelerations decreased with a decrease of propellant loading beginning at the 2000 lb load case. Higher load cases (at 4000 lb for example) exhibited decreased induced velocity due to decreased sloshing as the liquid free surface decreased within the spherical tank.

______________________, Committee Chair
Prof Jose Granda

______________________
Date
ACKNOWLEDGEMENTS

First, I would like to thank Professor Jose Granda for all of his academic guidance and assistance in the writing of this thesis. The weekly meetings in his office have been a valuable experience in gaining knowledge in the field of Dynamics.

Secondly, I am grateful to Louis Nguyen at NASA’s Johnson Space Center for taking much time out of his busy schedule to work with Dr. Granda and I on this work. The direction and information provided by Louis has been invaluable to this paper.
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CHAPTER 1
INTRODUCTION

1.1 Background

Many types of flight vehicles carry large quantities of liquid propellant. The relative motion of the liquid within the propellant tanks can be a significant source of disturbance in the control of a vehicle’s attitude and trajectory. An understanding of slosh dynamics and the resultant effects on the rigid body dynamics of the vehicle can be critical to mission success. One such example was in the failure of the Falcon 1 rocket in March 2007. Propellant slosh in the liquid oxygen (LOx) tank within the second stage created an oscillation, putting the stage in an uncontrollable roll, and eventually preventing propellant from reaching the engine\(^1\). Another example was the NEAR (Near Earth Asteroid Rendezvous) spacecraft sent to intercept the asteroid Eros in 1998. A routine orbital correction created an unexpected motion causing the spacecraft to enter safe mode. During this event, the entire propellant margin was spent as the thrusters fired thousands of times and resulted in a 13-month mission delay. Propellant slosh was identified as a probable cause\(^2\). Fortunately, the spacecraft was recovered and went on to complete a successful mission. A final example was seen during the Apollo 11 mission which was the first lunar landing in 1969, where sloshing of the propellant induced an oscillatory motion of the Lunar Module (Visually observed in the video footage), which
hampered accurate control of the landing maneuver. Accurate prediction of the effects of propellant slosh on the system can prevent mission anomalies such as these, saving money, time and possibly lives.

1.2 PROPELLANT SLOSH SUPRESSION
Several types of propellant slosh suppression devices have been included into the design of aerospace vehicles, examples of which include but are not limited to rigid ring baffles, flexible flat ring baffles, cruciform baffles, deflectors, and diaphragms. The effectiveness of each type of suppression device is typically expressed as the amount of damping provided to the propellant motion. This damping factor is then input into the analysis models in order to simulate slosh dynamics. This paper concentrates on rigid ring baffles and cruciform baffles as applied on the development lander. Rigid flat ring baffles (figure 1.2-1) are oriented in a horizontal orientation within the tank, and are effective at providing damping within the slosh motion of the liquid. The cruciform baffles shown in figure 1.2-1 utilize the structural stringers on the inside of the tank to use as attach points for the flat ring baffles but do not contribute a significant amount to damping. The dynamic simulation presented in this paper will model a rigid flat ring baffle design within the spherical propellant tanks.
Both rigid flat non-perforated baffles and perforated cruciform baffles have been studied since the 1960s\(^4\). Testing done at NASA concluded that horizontal ring baffles in spherical tanks provides a vastly higher reduction in slosh force than does the cruciform arrangement\(^4\). Comparison studies between single ring baffles and three ring baffles have revealed that the slosh forces were most significantly suppressed when the surface of the liquid was level with or slightly above the ring\(^4\). Experiments using a single rigid ring centered inside the tank exhibited ineffective damping at liquid levels more than 0.3 to 0.4 times the tank radius\(^4\). The three ring baffle was therefore more effective in...
suppressing liquid slosh where liquid level is not constant. The tank model studied in this analysis utilizes four horizontal solid rings to be effective at all propellant heights.

Fig. 1.2-2 Single Ring and Multi-Ring Baffle Configurations.
[Image taken from reference 4]

1.3 PROBLEM STATEMENT

NASA is currently in development of a vertical test bed vehicle demonstrating new green propellant propulsion systems and autonomous landing and hazard detection technology\(^6\). The vehicle will be required to land on the Earth’s surface without the aid of parachutes using a single gimbaled LOX/Methane engine to control its descent. One of the many challenges of this system is the ability to control the descent while the vehicle is exposed to external and internal disturbances, and under changing mass properties (propellant consumption). In order to develop a suitable control system, the magnitude of the disturbance inputs must be predicted. One such disturbance is the propellant slosh
created during vehicle accelerations. This paper analyzed the effects of propellant motion on the rigid body motion of a planetary lander for both liquid methane (propellant) and liquid oxygen (oxidizer) residing in spherical propellant tanks containing cruciform and rigid ring baffles, and at different levels of propellant loading. The analysis will predict the contribution of the rotation rates about the three body axes imparted by the propellant slosh. The results are discussed in Chapter 3.
CHAPTER 2
ANALYSIS TECHNIQUE

2.1 CAD MODEL DESCRIPTION: LANDER

The geometry of the NASA designed vehicle was originally modeled and supplied by NASA’s Johnson Space Center. The lander comprises of a central frame supporting four spherical propellant tanks, and a propulsion system. The assembly consisted of over 3000 components held together by fasteners, fittings, clamps etc… bringing the file size to over 35 MB. The problem associated with importing this CAD model directly into the SimWise 4D program was two-fold. First, the file size alone was too large for the software/computing machine to handle smoothly (best if under 10 MB). Secondly, the part count made the entire system a cumbersome model. All of the component connections needed to be understood within the SimWise 4D software. SimWise 4D understands how each individual component should behave through a set of assigned constraints given to each part. If no constraints are given, it uses the imported geometry coupled with collision detection to restrain components from flying apart. The calculation time required to determine the behavior off all parts, particularly with collision detection proved impractical. In order to provide SimWise 4D with a model that was “analysis friendly”, the Pro/ENGINEER model provided by NASA was required to be modified.
Fig. 2.1-1 Front View of Lander, NASA Supplied Pro/Engineer Model

Fig. 2.1-2 Top View of Lander, NASA Supplied Pro/Engineer Model
To create a new analysis model from the CAD model, all of the small components were removed, and the remaining components were fused together such that the 3000 plus parts would become a single rigid body structure. This would not affect the dynamic simulation of the vehicle in SimWise 4D because the software allows custom input of the inertia tensor and center of mass location independent of vehicle geometry. The output of the analysis is therefore unaffected by the geometric changes, as the inertia tensor and center of mass location will match the actual properties of the test vehicle. This successfully allowed a more friendly analysis model to be incorporated without sacrificing accuracy. Once the body was simplified, the components were then fused together again resulting in a single rigid body structure. The baffles inside the tank were removed to allow free motion of the separate propellant bodies within the tank.

Figure 2.1-3 NASA Supplied Model Vs. Simplified Lander Model
Figure 2.1-4 Model Simplification Flow Chart

1. **NASA Provided ProEngineer Model**
2. **Convert to .STEP file**
3. **Import .STEP into NX6 CAD software**
4. **Delete all minor components**
5. **Combine all remaining components into single part**
6. **Import into Simwise 4D**
Figures 2.1-4 and 2.1-5 show the simplification process between the initial system model and the simplified analysis model. Further simplification was done within the propellant tanks themselves. The support stringers and baffles were removed from the inside of the tank. These model components were not needed because the damping effect on the propellant by the baffles can be simulated by frictional damping within the propellant body constraints. This will be discussed in detail in section 2.7.
Figure 2.1-6 shows the inner tank geometry used in the analysis model.

The simplified “analysis model” reduced the file size from an original 93 MB to 5.2 MB including a representative propellant body included within each propellant tank. This greatly improved performance in terms of calculation time and motion visualization within SimWise 4D.

Figure 2.1-6 Tank Interior Modifications – Baffle Removal
2.2 CAD MODEL DESCRIPTION: PROPELLANT BODIES

The propellant for the lander system comprises of liquid methane (LCH$_4$) fuel residing in two of the four spherical tanks, and a liquid oxygen oxidizer (LOX) residing in the other two. The simulation modeled the liquid propellant as solid spherical sections, but restricted to a pendulum type motion around a specifically placed attach point. Details of how the physics was modeled is presented in section 2.3.

![Propellant Body](image-url)

Figure 2.2-1 Example of Solid Spherical Section Representing Propellant Body
In addition to geometric and mass property data, NASA supplied multiple propellant load cases in which the simulation was to model. The idea was that as fuel and oxidizer were consumed, the model would simulate the dynamic behavior of the propellant and lander at different propellant loadings (1000lbs, 900 lbs., 800 lbs., etc…). Therefore, the propellant slosh and rigid body motion would be predicted at different snapshots in time. The result would give insight into not only the change in dynamics of the propellant as
the propellant height within the tank decreased, but how the magnitude of the effect on the overall rigid body motion of the lander would change as the propellant height decreases.

Ideally, the mass and volume of the propellant bodies would both change at an inputted rate as the simulation is run, and would therefore not require multiple simulations at separate snapshots in time. Unfortunately, although the software allows input of a time function for mass, the volume input of the propellant bodies is fixed. Without the volume of the propellant body decreasing with the mass, the center of gravity of each propellant body would not travel towards the bottom of the tank as required, resulting in error.

Given a 1.8 mixture ratio (NASA supplied) between the LOx and LCH₄ quantities, the mass quantity for each propellant tank is broken down as follows:

<table>
<thead>
<tr>
<th>Total Load lb</th>
<th>Total Load, kg</th>
<th>LOx, kg</th>
<th>LCH₄, kg</th>
<th>LOx per tank, kg</th>
<th>LCH₄ per tank, kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>1814.369</td>
<td>1166.380</td>
<td>647.989</td>
<td>583.190</td>
<td>323.995</td>
</tr>
<tr>
<td>2000</td>
<td>907.185</td>
<td>583.190</td>
<td>323.995</td>
<td>291.595</td>
<td>161.997</td>
</tr>
<tr>
<td>1000</td>
<td>453.592</td>
<td>291.595</td>
<td>161.997</td>
<td>145.798</td>
<td>80.999</td>
</tr>
<tr>
<td>800</td>
<td>362.874</td>
<td>233.276</td>
<td>129.598</td>
<td>116.638</td>
<td>64.799</td>
</tr>
<tr>
<td>600</td>
<td>272.155</td>
<td>174.957</td>
<td>97.198</td>
<td>87.479</td>
<td>48.599</td>
</tr>
<tr>
<td>400</td>
<td>181.437</td>
<td>116.638</td>
<td>64.799</td>
<td>58.319</td>
<td>32.399</td>
</tr>
<tr>
<td>100</td>
<td>45.359</td>
<td>29.160</td>
<td>16.200</td>
<td>14.580</td>
<td>8.100</td>
</tr>
</tbody>
</table>

Table 2.2-1 Propellant Load Cases

These load cases give seven examples of the snapshots in time to be simulated for model validation of the composite lander/propellant system. For example in the 1000 lb. load
case, the table shows that there will be 145.798 kg of LOx in each of the liquid oxygen tanks, and 80.999 kg in each of the liquid methane tanks, for a total mass loading of (2 x 145.798) + (2 x 80.999) = 453.592 kg (1000 lb).

Given the mass at each propellant level, the first step is to determine the corresponding propellant height within each tank. This would result in an accurate center of mass position for each propellant body.

Full derivation of the propellant moments of inertia, center of mass, and propellant height within spherical tanks can be found in reference 7.

![Figure 2.2-3 Propellant Height Diagram of Propellant Within Tank.](image)

Image taken from Reference 7.

The height “h” of the propellant is shown in figure 2.2-3 relative to the geometric quantities of the tank. The height “h” is measured from the bottom of the tank, and is a
function of the tank inner radius “R”, the propellant mass “m”, and the density of the propellant, ρ. Once the height of the propellant has been determined, it can be input into a new propellant body model in the NX8 CAD program and subsequently incorporated into the SimWise 4D model as a neutral file.

If the tank was completely full, the full tank mass “m₀” would be:

\[ m₀ = \rho_{Fuel}V_{Tank} = \frac{4}{3} \rho \pi R^3 \]  \hspace{1cm} (2.2-1)

The propellant volume of a partially filled tank can be found by integrating an infinitesimally thin disk of radius \( R_d \) over the height of the propellant (see figure 2.2-1). \( R_d \) is seen to be:

\[ R_d = \sqrt{R^2 - x^2} \]  \hspace{1cm} (2.2-2)

The volume of the disk is computed as the area multiplied by the thickness:

\[ V_d = \pi R_d^2 dx \]  \hspace{1cm} (2.2-3)

Integrating to obtain the propellant volume, the result is:
The mass of the propellant is therefore:

\[
V_{\text{fuel}} = -\frac{R-h}{R} \frac{\pi R^2 - x^2}{x} \, dx = \frac{\pi h^2}{3} \left(3R - h\right)
\]  

(2.2-4)

\[
m_{\text{fuel}} = \frac{\pi h^2}{3} \rho \left(3R - h\right)
\]  

(2.2-5)

Equation 2.5-5 can be written in terms of propellant height as:

\[
h^3 - 3Rh^2 + \frac{3m_{\text{fuel}}}{\pi \rho} = 0
\]  

(2.2-6)

The solution to the cubic polynomial is:

\[
h = 1 - 2 \cos \left(\frac{\pi + \theta}{3}\right) R
\]  

(2.2-7)

Where,

- \(h\) = propellant height from bottom of tank
- \(R\) = Tank inner radius
- \(\theta = \cos^{-1} 1 - 2 \xi\)
- \(\xi = \frac{m_{\text{fuel}}}{m_0}\)
1000 lb. Propellant Load Case: LOx

\[ m_0 = \frac{4}{3} \pi \, 1141 \, 0.61^3 = 1084.8 \, kg \]  

(2.2-8)

For a LOx loading of 145.798 kg:

\[ \theta = \cos^{-1} \, 1 - 2 \, \frac{145.798 \, kg}{1084.8 \, kg} = 0.757 \, radians \]  

(2.2-9)

Therefore the LOx height in each oxidizer tank is found to be:

\[ h_{LOx} = 1 - 2 \cos \, \frac{\pi + 0.757}{3} \, 0.61 \, m = 0.284 \, m \]  

(2.2-10)

Similarly, for the 1000 lb. Propellant Load Case: LCH\(_4\)

\[ m_0 = \frac{4}{3} \pi \, 465 \, 0.61^3 = 442.11 \, kg \]  

(2.2-11)

For a LCH\(_4\) loading of 80.999 kg:

\[ \theta = \cos^{-1} \, 1 - 2 \, \frac{80.999 \, kg}{442.11 \, kg} = 0.866 \, radians \]  

(2.2-12)
Therefore the LCH₄ height in each propellant tank is found to be:

\[ h_{LCH_{4}} = 1 - 2 \cos \left( \frac{\pi + 0.866}{3} \right) \quad 0.61 \text{ m} = 0.326 \text{ m} \]  

(2.2-13)

With the propellant heights determined for each tank at the 1000 lb. load level, the revised propellant bodies can be modeled with little effort. A section cut through a solid sphere at the calculated height will result in the propellant body section, which will fill approximately 1/4 of the tank for both the liquid methane and liquid oxygen.
As shown in Table 2.2-1, the mix ratio of LOx oxidizer to LCH₄ propellant is 1.8. However, the difference in propellant heights between the LOx and liquid methane is only .042 meters. Although almost twice as much oxidizer is used compared to the liquid methane propellant, the height of the liquid methane is slightly higher. The reason for this is that the density of liquid oxygen is 2.45 times higher than liquid methane.

800 lb. Load Case: LOx

\[ m_0 = \frac{4}{3} \pi \times 1141 \times 0.61^3 = 1084.8 \text{ kg} \]  \hspace{1cm} (2.2-14)

For a LOx loading of 116.64 kg (Table 2.5-1):

\[ \theta = \cos^{-1} \left( 1 - 2 \frac{116.64 \text{ kg}}{1084.8 \text{ kg}} \right) = 0.676 \text{ radians} \]  \hspace{1cm} (2.2-15)

Therefore the LOx height in each oxidizer tank is found to be:

\[ h_{LOx} = 1 - 2 \cos \frac{\pi + 0.676}{3} = 0.61 \text{ m} = 0.251 \text{ m} \]  \hspace{1cm} (2.2-16)

With the propellant heights determined for each tank at the 800 lb. load level, the revised propellant bodies can be modeled. Again, a section cut through a solid sphere at the
calculated height of .251 m will result in the propellant body section, which can then be fed into the model.

800 lb. Propellant Load Case: LCH\textsubscript{4}

\[ m_0 = \frac{4}{3} \pi \ 465 \ 0.61^3 = 442.11 \text{ kg} \]  \hspace{1cm} (2.2-17)

For a LCH\textsubscript{4} loading of 62.29 kg (Table 2.5-1):

\[ \theta = \cos^{-1} (1 - 2 \times \frac{64.79 \text{ kg}}{442.11 \text{ kg}}) = 0.770 \text{ radians} \]  \hspace{1cm} (2.2-18)

Therefore the LCH\textsubscript{4} height in each propellant tank is found to be:

\[ h_{LCH_4} = 1 - 2 \cos \frac{\pi + 0.770}{3} \ 0.61 \text{ m} = 0.297 \text{ m} \]  \hspace{1cm} (2.2-19)

600 lb. and 100 lb. LOx and LCH\textsubscript{4} Load Cases

Similarly, the 600 lb. and 100 lb. LOx and LCH\textsubscript{4} propellant heights can be found.

For the 600 lb. load case:

\[ h_{LOX} = 0.215 \text{ m} \]  \hspace{1cm} (2.2-20)
\( h_{LCH_4} = 0.254 \, m \) \hspace{1cm} (2.2-21)

For the 100 lb. Load Case:

\( h_{LOx} = 0.084 \, m \) \hspace{1cm} (2.2-22)

\( h_{LCH_4} = 0.098 \, m \) \hspace{1cm} (2.2-23)

A table of resulting propellant loadings with propellant heights and CG location relative to the body coordinates is shown in table 2.2-2.

<table>
<thead>
<tr>
<th>Loading, lbs</th>
<th>( h ) (LOx), m</th>
<th>( h ) (LCH4), m</th>
<th>CG_X: LOx</th>
<th>CG_X: LCH4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.647</td>
<td>0.816</td>
<td>1.248</td>
<td>1.336</td>
</tr>
<tr>
<td>2000</td>
<td>0.419</td>
<td>0.504</td>
<td>1.115</td>
<td>1.166</td>
</tr>
<tr>
<td>1000</td>
<td>0.284</td>
<td>0.326</td>
<td>1.029</td>
<td>1.055</td>
</tr>
<tr>
<td>800</td>
<td>0.251</td>
<td>0.297</td>
<td>1.010</td>
<td>1.040</td>
</tr>
<tr>
<td>600</td>
<td>0.215</td>
<td>0.254</td>
<td>0.987</td>
<td>1.012</td>
</tr>
<tr>
<td>400</td>
<td>0.173</td>
<td>0.204</td>
<td>0.960</td>
<td>0.980</td>
</tr>
<tr>
<td>100</td>
<td>0.084</td>
<td>0.098</td>
<td>0.902</td>
<td>0.911</td>
</tr>
</tbody>
</table>

Table 2.2-2 Propellant Heights and C.G. Locations
2.3 COMPOSITE ANALYSIS MODEL DESCRIPTION

Both the dry lander (no propellant) and the propellant bodies described in sections 2.1 and 2.2 respectively were combined to form a composite model of the rigid body system. The origin of the coordinate system was located within the plane making up the bottom of the lander “feet”, and geometrically centered between the four lander feet.

![Figure 2.3-1 Composite Analysis Model](image)

The coordinate system is as shown in figure 2.3-1. The positive x-axis oriented “upward” opposing gravity, and the positive z coordinate passes through the center of LOX Tank #1. The SimWise 4D program was adjusted to align with this orientation.
The coordinate system was chosen in order to align the orientation with NASA’s existing models. This allowed a direct comparison of analysis results without requiring a transformation.

Figure 2.3-2 World Orientation

With the coordinate system and world orientation aligned with NASA’s model, and the physical bodies modeled in CAD, the next step was to setup the boundary conditions within SimWise 4D in order to simulate the propellant motion of each independent propellant body, and the rigid body motion of the lander.
2.4 MECHANICAL SLOSH MODEL

The slosh dynamics were modeled using a pendulum model using the solid spherical sections as the pendulum masses. Each liquid body was allowed to rotate freely about a carefully chosen pendulum attach point. To restrict the propellant bodies to 3D rotation about the pendulum attach point, a spherical joint constraint was used. The spherical joint in SimWise 4D allows only rotation about the joint location with no translation. The spherical joint constraint was made fixed relative to the lander body (such that it always remained at the chosen point relative to the propellant tank. A physical pendulum arm was not required to be modeled under this constraint definition as the spherical joint is virtually attached to the liquid body center of mass, equivalent to a virtual massless arm. If the pendulum arm needed to be adjusted, this could be accomplished by moving the spherical joint closer to or farther away from the propellant body. Figure 2.4-1 shows the spherical joint restraint on a propellant body.
Figure 2.4-1 Spherical Joint Constraint

Figure 2.4-2 Propellant Body Rotation About Spherical Joint
Since the forces imparted on the lander by the swinging pendulum mass (propellant) are exerted at the pendulum attach point, the attach point needs to be located at the propellant body center of mass within the propellant tank. In addition, the pendulum arm length needs to be properly chosen to achieve the correct frequency that most closely simulates the liquid propellant slosh within the tanks. The pendulum arm length was therefore chosen based on the desired natural frequency of the LCH₄ and LOx bodies.

The natural frequency of liquid motion in different shaped tanks was studied by the Southwest Research Institute of Texas, which also drew upon heritage studies done on liquid motion within moving bodies⁴. The paper provided a theoretical natural frequency for the pendulum motion using the SLOSH computer code to model the liquid inside a spherical tank as a function of liquid height, and tank radius.

Figure 2.4-3 Natural Frequency for Liquid in Spherical Tanks (Reference 5)
LOx and LCH$_4$ Theoretical Natural Frequency: 1000 lb. Propellant loading

Looking at the first mode, the theoretical natural frequency for each LOx body can be obtained:

\[ R_0 = 0.61 \text{ m} \text{ (Radius of Tank)} \]

\[ h = 0.284 \text{ m} \text{ (From Table 2.2-2)} \]

For an h/R$_0$ value of 0.466, figure 2.3-3 predicts a non-dimensional natural frequency parameter of approximately 1.09.

\[ \omega_n \frac{R_0}{g}^{0.5} = 1.09 \quad (2.4-1) \]

\[ \omega_n \frac{0.61 \text{ m}}{9.81 \text{ m/s}^2}^{0.5} = 1.09 \quad (2.4-2) \]

\[ \omega_n = 4.371 \text{ rad/sec} \quad (2.4-3) \]

\[ f_n = \frac{\omega_n}{2\pi} \quad (2.4-4) \]

\[ f_{n,LOx} = \frac{4.371 \text{ rad/sec}}{2\pi} = 0.696 \text{ Hz} \quad (2.4-5) \]
This is the predicted value for the LOx propellant body residing in a 0.61 m spherical tank. The natural frequency for each LCH4 body can be obtained in a similar manner:

\[ R_0 = 0.61 \, m \]
\[ h = 0.326 \, m \, (\text{From 2.5-13}) \]

For an \( h/R_0 \) value of 0.534, Figure 2.4-1 predicts a non-dimensional natural frequency parameter of 1.11.

\[ \omega_n = 4.411 \, \text{rad/sec} \]  
\[ (2.4.8) \]

\[ f_n = \frac{\omega_n}{2\pi} \]  
\[ (2.4.9) \]

\[ f_{n,LCH4} = \frac{4.411 \, \text{rad/sec}}{2\pi} = 0.702 \, \text{Hz} \]  
\[ (2.4.10) \]

The frequency for the remaining load cases can similarly be obtained, and are shown in table 2.4-1.
Table 2.4-1 Pendulum Frequencies

<table>
<thead>
<tr>
<th>Loading, lbs</th>
<th>Frequency (LOx), Hz</th>
<th>Frequency (LCH4), Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.811</td>
<td>0.894</td>
</tr>
<tr>
<td>2000</td>
<td>0.718</td>
<td>0.766</td>
</tr>
<tr>
<td>1000</td>
<td>0.696</td>
<td>0.702</td>
</tr>
<tr>
<td>800</td>
<td>0.683</td>
<td>0.696</td>
</tr>
<tr>
<td>600</td>
<td>0.670</td>
<td>0.683</td>
</tr>
<tr>
<td>400</td>
<td>0.657</td>
<td>0.670</td>
</tr>
<tr>
<td>100</td>
<td>0.638</td>
<td>0.651</td>
</tr>
</tbody>
</table>

Note that as the propellant loading decreases, the natural frequency of the propellant slosh also decreases.

Pendulum Length

With the theoretical natural frequency obtained as shown in table 2.4-1, the required pendulum length can then be determined by the following:

\[
L = \frac{g}{4\pi^2 f^2}
\]  
(2.4-11)

For the 1000 lb. LOx load case:

\[
L = \frac{9.81 \frac{m}{s}}{4\pi^2 \cdot 0.702^2} = 0.504 \text{ m}
\]  
(2.4-12)

The pendulum lengths for the remaining load cases can similarly be found as shown above and are presented in table 2.4-2.
Table 2.4-2 Pendulum Lengths

To verify the frequency within the SimWise 4D Software, a test case was run using each load case.

<table>
<thead>
<tr>
<th>Loading, lbs</th>
<th>LOx Pendulum Length, m</th>
<th>LCH4 Pendulum Length, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.378</td>
<td>0.311</td>
</tr>
<tr>
<td>2000</td>
<td>0.482</td>
<td>0.424</td>
</tr>
<tr>
<td>1000</td>
<td>0.513</td>
<td>0.504</td>
</tr>
<tr>
<td>800</td>
<td>0.533</td>
<td>0.513</td>
</tr>
<tr>
<td>600</td>
<td>0.553</td>
<td>0.533</td>
</tr>
<tr>
<td>400</td>
<td>0.575</td>
<td>0.553</td>
</tr>
<tr>
<td>100</td>
<td>0.610</td>
<td>0.586</td>
</tr>
</tbody>
</table>

Figure 2.4-4 LOx Natural Frequency Simulation
The test case for the 1000 lb. total propellant load is shown in figure 2.4-4. A single LCH4 propellant body was hinged at a pendulum length of .504 meters and given an initial velocity of 0.5 m/s. The resulting data is shown in figure 2.4-5. The plot represents the position of the center of mass vs. time. The plot data was copied into an Excel spreadsheet and the natural frequency was extracted.

![Figure 2.4-5 Frequency Test with .504 Pendulum Length](image)

The data shows that the periodic motion of the LOx body has a period of 1.43 seconds. This translates into a frequency of:

\[
    f_{n,LOx} = \frac{1}{T} = \frac{1}{1.43} = 0.699 \text{ Hz}
\]  

Looking back at the theoretical value (Table 2.4-1) the predicted value was 0.702 Hz.
The percent difference is:

$$\text{% Difference} = \frac{0.699 - 0.702}{0.702} \times 100 = .42\%$$

(2.4-14)

The frequency test above confirms that the SimWise software program will accurately
implement the desired frequency needed to simulate the propellant slosh. As seen on the
theoretical chart (figure 2.4-3), some error originates in the approximation of where the
correct point on the curve resides on the Y-axis value for the natural frequency
parameter.

Once the slosh frequencies within the model were verified, the propellant bodies were
placed below the tank at the proper distance from the pendulum attach point. The masses
were defined to “penetrate all” such that no physical interference would be present.
Figure 2.4-6 LCH4 and LOX Mass Modeled Below Tank

As shown in figure 2.4-6, the propellant bodies are suspended below the tank. Although the graphical representation is inaccurate, the results will be accurate as the force of the pendulum masses will act at the pendulum attach point, which coincides with that “real” location of the propellant center of mass.
Figure 2.4-7 Propellant Bodies in Constrained Motion Within Tanks

Figure 2.4-8 shows an image of the actual test vehicle, dubbed “project Morpheus” during a tethered test using a crane.

Figure 2.4-8 Project Morpheus Tether Test [Taken from reference 6]
For this analysis, only the angular rates imparted on the lander by the propellant slosh was of interest. To restrict the vehicle translational motion, a spherical joint was also placed at the lander’s center of mass.

2.5 MASS PROPERTIES

SimWise 4D will automatically calculate the center of mass and inertia tensor of each body (about its center of mass) given a homogeneous mass. The analyst also has the ability to input custom properties such as mass, friction, inertia, and center mass location. This is useful because as the mass properties change in the actual test vehicle due to design updates, the model (from a visual geometric sense) does not require updating. If the new mass properties can be obtained, the new inertia tensor and center of mass location can be directly input into the model. SimWise 4D will calculate the solutions to the equations of motion using these input properties independent of the modeled geometry.

The densities of the liquid methane (LCH4) and liquid oxygen (LOx), along with the fuel/oxidizer ratios at different mass loadings were provided by NASA. The lander dry center of mass (no propellant) was provided to be:

<table>
<thead>
<tr>
<th>Mass, kg</th>
<th>975.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_CM, m</td>
<td>1.54</td>
</tr>
<tr>
<td>y_CM, m</td>
<td>0.00</td>
</tr>
<tr>
<td>z_CM, m</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2.5-1 Dry Lander Center of Mass Location
This was directly input into the SimWise 4D model as shown in figure 2.4-1.

The center of mass of each propellant body was calculated by SimWise given a mass input. Therefore the composite center of mass was a combination of custom and calculated inputs. The center of mass of the propellant body and the center of mass of the composite system were calculated by SimWise 4D. Only the dry vehicle mass properties were customized.
The next set of mass properties to assign to the system was the inertia tensor. The inertia tensor can be displayed in matrix form as the following:

\[
[I] = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix}
\]  

(2.5-1)

The first moments of inertia about the body axes can be written as:

\[
l_{xx} = x^2 dm = y^2 + z^2 dm
\]

(2.5-2)

\[
l_{yy} = y^2 dm = x^2 + z^2 dm
\]

(2.5-3)

\[
l_{zz} = y^2 dm = x^2 + z^2 dm
\]

(2.5-4)

The first moments make up the main diagonal of the inertia tensor. The off axis moments, or products of inertia are defined as follows:

\[
l_{xy} = xy dm = l_{yx}
\]

(2.5-5)

\[
l_{xz} = xz dm = l_{zx}
\]

(2.5-6)

\[
l_{yz} = yz dm = l_{zy}
\]

(2.5-7)

By default, the inertia tensor is calculated automatically about the body center of mass by SimWise 4D as shown in figure 2.4-2.
The inertia tensor is calculated assuming a uniform density throughout the body. NASA supplied the following inertia tensor for the dry lander.
Table 2.5-2 Inertia Tensor of Dry Lander about CM

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{xx}$</td>
<td>756.17</td>
<td></td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>697.57</td>
<td></td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>786.86</td>
<td></td>
</tr>
<tr>
<td>$I_{xy}$</td>
<td>-3.77</td>
<td></td>
</tr>
<tr>
<td>$I_{xz}$</td>
<td>-8.43</td>
<td></td>
</tr>
<tr>
<td>$I_{yz}$</td>
<td>-7.81</td>
<td></td>
</tr>
</tbody>
</table>

(inertia in kg-m^2)

Since the inertia tensor is a symmetric matrix, it follows that $I_{xy} = I_{yx}$, $I_{xz} = I_{zx}$, $I_{yz} = I_{zy}$.

The inertia tensor for the dry lander (no propellant) that is to be input into SimWise 4D is therefore:

$$[I]_{\text{Lander}} = \begin{bmatrix} 756.17 & -3.77 & -8.43 \\ -3.77 & 697.57 & -7.81 \\ -8.43 & -7.81 & 786.86 \end{bmatrix} \quad (2.5-8)$$

The products of inertia, namely $I_{xy}$, $I_{xz}$, $I_{yx}$, $I_{yz}$, $I_{zx}$ and $I_{zy}$ for this coordinate frame are non-zero quantities, and therefore the body axes of the vehicle do not align with the principal axes. This will be physically seen as a dynamic unbalance in the system as the mass distribution is not perfectly symmetrical about the axes. A moment about the $x$ axis, for example, will result in secondary rotation about the $y$ and $z$ axes as well which will be seen as a “wobble”. This secondary rotation will cause further propellant slosh
motion and additional input into the rotation rates of the vehicle. This secondary effect will be quantified in section 2.6.

It is seen that the moment of inertia tensor calculated by SimWise 4D in fig. 2.4-2 is not equal to the inertia tensor provided by NASA. This is due to geometric simplification of the analysis model as described earlier in this chapter, in addition to the assumption by the software that the single rigid body structure of the lander is homogeneous in density. Similar to how the center of mass was custom input into the SimWise 4D software, the moment of inertia tensor can be as well. This inertia tensor is given about the body axis originating from the dry (no propellant) lander’s center of mass. The input of the custom inertia tensor is shown in figure 2.4-3.

![Custom Inertia Tensor for Dry Lander](image)

Figure 2.5-3 Custom Inertia Tensor for Dry Lander
The density of the LOx and LCH4 was provided by NASA and is shown in table 2.4-3.

<table>
<thead>
<tr>
<th>rho_LOx (kg/m^3)</th>
<th>LCH4_rho (kg/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1141.00</td>
<td>465.00</td>
</tr>
</tbody>
</table>

Table 2.5-3 Liquid Oxygen and Liquid Methane Densities

With the center of mass and moments of inertia now aligned with the properties provided by NASA, the next step was to run simulations on the propellant slosh bodies and compare the slosh dynamics with other independent models. Achieving similar results was needed to validate the model prior to running full system simulations.

2.6 MODEL VALIDATION

In parallel to the SimeWise 4D simulation, slosh analysis was performed by NASA using the SOMBAT software code to predict the magnitude of the angular rates imparted on the vehicle. NASA suggested 13 test cases at the 1000 lb load level to perform in SimWise 4D for comparison with the SOMBAT results.

<table>
<thead>
<tr>
<th>TC</th>
<th>LOx-1</th>
<th>LOx-2</th>
<th>LCH4-1</th>
<th>LCH4-2</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1 kg</td>
<td>0.1 kg</td>
<td>LOx pendulum attch pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Z</td>
<td>0° Z</td>
<td>LCH4 pendulum attch pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>-15° Z</td>
<td>+15° Z</td>
<td></td>
<td></td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>2</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1 kg</td>
<td>0.1 kg</td>
<td>LOx pendulum attch pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Z</td>
<td>0° Z</td>
<td>LCH4 pendulum attch pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>+15° Z</td>
<td>+15° Z</td>
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<td></td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>3</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1 kg</td>
<td>0.1 kg</td>
<td>LOx pendulum attch pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Z</td>
<td>0° Z</td>
<td>LCH4 pendulum attch pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>-15° Z</td>
<td>-15° Z</td>
<td></td>
<td></td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
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<td>148.94 kg</td>
<td>0.1 kg</td>
<td>0.1 kg</td>
<td>LOx pendulum attch pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Z</td>
<td>0° Z</td>
<td>LCH4 pendulum attch pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>+15° Z</td>
<td>+15° Z</td>
<td></td>
<td></td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>2a</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1 kg</td>
<td>0.1 kg</td>
<td>LOx pendulum attch pt., X = 1.5367</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Z</td>
<td>0° Z</td>
<td>LCH4 pendulum attch pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>+15° Z</td>
<td>+15° Z</td>
<td></td>
<td></td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>2b</td>
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<td>148.94 kg</td>
<td>0.1 kg</td>
<td>0.1 kg</td>
<td>LOx pendulum attch pt., X = 1.4560</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Z</td>
<td>0° Z</td>
<td>LCH4 pendulum attch pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>+15° Z</td>
<td>+15° Z</td>
<td></td>
<td></td>
<td>Pendulum length 0.25865</td>
</tr>
</tbody>
</table>

All SI units (kg, meter)
Table 2.6-2 Test Cases 5 thru 7

<table>
<thead>
<tr>
<th>TC</th>
<th>LOx-1</th>
<th>LOx-2</th>
<th>LCH4-1</th>
<th>LCH4-2</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1kg</td>
<td>0.1kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>+15° Y</td>
<td>-15° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>Pendulum length 0.25865</td>
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<tr>
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<td>0.1kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>+15° Y</td>
<td>+15° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>7</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1kg</td>
<td>0.1kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>-15° Y</td>
<td>-15° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
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<td>6a</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>0.1kg</td>
<td>0.1kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>+15° Y</td>
<td>+15° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>Pendulum length 0.25865</td>
</tr>
</tbody>
</table>

All SI units (kg, meter)

Table 2.6-3 Test Case 8 & Special Cases 1M and 2M

<table>
<thead>
<tr>
<th>TC</th>
<th>LOx-1</th>
<th>LOx-2</th>
<th>LCH4-1</th>
<th>LCH4-2</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1M</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>77.86 kg</td>
<td>77.86 kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>-15° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>2M</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>77.86 kg</td>
<td>77.86 kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>+15° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>0° Z</td>
<td>Pendulum length 0.25865</td>
</tr>
<tr>
<td>8</td>
<td>148.94 kg</td>
<td>148.94 kg</td>
<td>77.86 kg</td>
<td>77.86 kg</td>
<td>LOx pendulum attach pt., X = 1.03467</td>
</tr>
<tr>
<td></td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>0° Y</td>
<td>LCH4 pendulum attach pt., X = 1.06131</td>
</tr>
<tr>
<td></td>
<td>+15° Z</td>
<td>+15° Z</td>
<td>0° Y</td>
<td>0° Y</td>
<td>Pendulum length 0.25865</td>
</tr>
</tbody>
</table>

All SI units (kg, meter)

Test cases 1, 2, 2a, and test case 6 are shown below for comparison.

Test Case 1

The objective of test case 1 was to measure the vehicle response with each LOx propellant body rotated about the Y axis 15 degrees in opposite directions, see figure 2.6-1.
Figure 2.6-1 Test Case 1, LOx bodies at +/- 15 deg about y axis

As seen in figure 2.6-1, the spherical joint serving as the pendulum attach point is located within the tank at the location where the propellant center of mass would be located. Each test case was run for 60 seconds. The LCH4 bodies (to the left and right of the image) were given 0.1 kg mass such that the effects of the opposed LOx bodies (forward and aft in figure 2.6-1) could be assessed. The results of the SimWise 4D simulation are shown in figure 2.6-2. The NASA SOMBAT simulation results are shown in figure 2.6-3.
Figure 2.6-2 Test Case 1 SimWise 4D Result. Lander Angular Rates.

Figure 2.6-3 Test Case 1 NASA SOMBAT Result. Lander Angular Rates
Comparing the NASA SOMBAT result and the SimWise 4D result it can be seen that both analysis results are in agreement. The two 148.94 kg LOx bodies opposed at +/- 15 degrees about the y axis creates approximately 5 deg/s of angular rotation about vertical x axis (roll).

**Test Case 2**

The objective of test case 2 was to measure the vehicle response with each LOx propellant body rotated about the y-axis 15 degrees in the same directions (+y axis), see figure 2.6-7.

![Figure 2.6-4 Test Case 2a. SimWise 4D Model](image-url)
The simulation was run for 60s and compared to the NASA SOMBAT simulation. The SimWise 4D results are shown in figure 2.6-5.

![Figure 2.6-5 Angular Rates for Test Case 2. SimWise 4D Result.](image)

The propellant body configuration results in a lander angular rate between approximately 1 and 4 deg/s about the z axis. This result from SimWise 4D shows a slightly more severe disturbance compared with the result from NASA’s SOMBAT program, which
predicts a max angular rate of approximately 3 deg/s about the Z axis. SOMBAT result is shown in figure 2.6-6.

![Pendulum Slop Test TC2: Vehicle Angular Rates](image)

Figure 2.6-6 Lander Angular Rates for Test Case 2. SOMBAT Result.

**Test Case 2a**

As described in the test matrix shown in figure 2.6-1, the objective of test case 2a was to place the pendulum attach point at equal height of the lander center of mass location.

Theoretically, there should be no rotation imparted on the lander, as the forces from the pendulum mass are passing through the lander body center of mass. The model for test case 2a is shown in figure 2.6-8.
Figure 2.6-7 Test Case 2a. SimWise 4D Model.

Figure 2.6-8 Lander Angular Rates for Test Case 2a. SimWise 4D Result.
As expected, the propellant slosh results in purely translational motion when the center of mass of the propellant (modeled as pendulum attach point) coincides with the lander center of mass. The angular rates shown in figure 2.6-9 are zero. The result agrees with the analysis performed by NASA using the SOMBAT code. The SOMBAT results are shown below in figure 2.6-10.

![Pendulum Slosh Test TC2a: Vehicle Angular Rates](image)

Figure 2.6-9  Lander Angular Rate for Test Case 2a. SOMBAT Result.

**Test Case 6**

As described in the test matrix shown in figure 2.6-2, the objective of test case 6 was essentially the same as test case 2, but with the LOx propellant bodies rotated about the
y-axis as opposed to the Z axis. This would identify differences as the vehicle rotates about a different set of axes. The model for test case 6 is shown in figure 2.6-11.

As shown in figure 2.4-10 the two LOx bodies are rotated about the y-axis about the spherical joint. The results of the SimWise 4D simulation are shown in figure 2.6-12.
As shown in figure 2.6-19, the angular rates on the lander produced by the slosh forces are reduced by approximately 1-1.5 deg/s compared to the rates seen in Test Case 2. Although the dry lander has higher inertia about the z axis (test case 2), test case 2 results in higher angular rates due to the LOx body position. The LOx bodies are farther away from the axis of rotation in Test Case 6 (y-axis) resulting in higher system inertia compared to test case 2. The SOMBAT results performed by NASA are shown in figure 2.6-12 for comparison.
As shown in Figure 2.6-12, the SOMBAT results show lander rotation about the pitch (y-axis) in the range of approximately 1.8 – 2.5 deg/sec. The results show not only a less severe induced rotation, but a smaller reduction compared to SOMBAT test case 2 results.

The results of the SimWise 4D simulation can be expected to be slightly more conservative than the SOMBAT simulation based on the above test cases.
2.7 Damping

The above validation test cases were run under ideal conditions (no losses) to test the lander response to propellant slosh inputs. However as the liquid propellant sloshes within the spherical tank, the energy dissipation created by the inner walls of the tank as well as the installed baffles will diminish the corresponding effect on the lander motion over time. This damping effect must be accounted for to achieve an accurate simulation. Baffles are relatively ineffective in altering the slosh frequency within the tank when the baffle resides away from the free surface\textsuperscript{5} and therefore the values calculated in table 2.2-3 are valid.

Without anti-sloshing devices such as baffles, damping occurs due to the viscous stresses and are determined experimentally. For spherical tanks, the damping has been found to following figure 2.7-1\textsuperscript{5}.

![Figure 2.7-1 Viscous Damping](image)
Since the spherical tank for this study contains four rigid ring baffles, the damping contribution from the baffles are much greater than that added by the viscous stresses, and therefore the latter contribution can be ignored. The below theory is derived from a single rigid ring baffle. The combined damping of all four baffles cannot assumed to be additive and would need to be determined through experimentation. For conservatism, the damping for a single ring baffle will be used as a minimum expected damping. For the 4000 lb case, Baffle #2 was utilized with the other baffles ignored for conservatism. For all other load cases, baffle #1 was used due to the liquid height within the tank.

The damping ratio is defined as:

\[
\gamma = \ln \left( \frac{\text{Maximum Amplitude}}{\text{Maximum Amplitude of next Cycle}} \right)
\]

(2.7-1)

The damping coefficient of a flat ring baffle within a cylindrical tank can be calculated by the following equation\(^5\):

\[
\gamma = \frac{15}{3\pi} \frac{4^2}{2} C_1 A f d^{2.5} \frac{\delta W}{\delta} \frac{m_s}{\rho} r^2
\]

(2.7-2)

Where,

\(C_1 = \text{(baffle area / tank cross-sectional area)}\)
A = tank cross-sectional area

\( f_d \) = depth function = (Amplitude at baffle location / wave height)

\( \delta \) = wave height

w = baffle width = .1016 m

\( m_o \) = mass of oscillator (propellant mass)

\( \rho \) = density

\( \Gamma \) = Max lateral oscillatory amplitude

Figure 2.7-2. Schematic for rigid ring baffle in cylindrical tank. Image taken from reference 5.

Although equation 2.7-2 was derived for a cylindrical tank, the equation actually applies to any tank shape. Therefore the damping for a rigid ring baffle in the current spherical tank can be predicted by substituting a depth function \( f_d \) and slosh mass that are specific for the spherical tank and baffle geometry used in this study. The tank studied in this
paper contains four rigid ring baffles as shown in figure 2.7-2. The magnitude of the damping coefficient will depend on the propellant height and relative location of the baffle used ($h_s$). The baffles will be identified as baffle 1 through 4 as shown in figure 2.7-2.
The baffle cross sectional areas are the following:

Baffle 1: 0.2121 m$^2$
Baffle 2: 0.3317 m$^2$
Baffle 3: 0.3415 m$^2$
Baffle 4: 0.3007 m$^2$

Tank cross sectional area at each baffle is the following:

Cross-section at Baffle 1: A = 0.5107 m$^2$
Cross-section at Baffle 2: A = 1.0958 m$^2$
Cross-section at Baffle 3: A = 1.1453 m$^2$
Cross-section at Baffle 4: A = 0.9097 m$^2$

Therefore, the $C_1$ for equation 2.6-1 at each baffle location is the following:

Baffle 1, $C_1 = 0.415$
Baffle 2, $C_1 = 0.302$
Baffle 3, $C_1 = 0.298$
Baffle 4, $C_1 = 0.331$

The wave height, $\delta$ is shown below in figure 2.7-3.
Table 2.7-1 shows the different wave heights for the various propellant load levels.

The baffle width for the 4 baffles is:

\[ w = 0.1016 \text{ m} \]
The mass of the propellant, \( m_s \), is shown below in table 2.7-2.

<table>
<thead>
<tr>
<th>Total Load lb</th>
<th>Total Load, kg</th>
<th>LOx, kg</th>
<th>LCH4, kg</th>
<th>LOx per tank</th>
<th>LCH4 per tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>1814.369</td>
<td>1166.380</td>
<td>647.989</td>
<td>583.190</td>
<td>323.995</td>
</tr>
<tr>
<td>3000</td>
<td>1360.777</td>
<td>874.785</td>
<td>485.992</td>
<td>437.393</td>
<td>242.996</td>
</tr>
<tr>
<td>2000</td>
<td>907.185</td>
<td>583.190</td>
<td>323.995</td>
<td>291.595</td>
<td>161.997</td>
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<td>453.592</td>
<td>291.595</td>
<td>161.997</td>
<td>145.798</td>
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<td>129.595</td>
<td>116.638</td>
<td>64.799</td>
</tr>
<tr>
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<td>272.155</td>
<td>174.957</td>
<td>97.198</td>
<td>87.479</td>
<td>48.599</td>
</tr>
<tr>
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<td>181.437</td>
<td>116.638</td>
<td>64.799</td>
<td>58.319</td>
<td>32.399</td>
</tr>
<tr>
<td>100</td>
<td>45.359</td>
<td>29.160</td>
<td>16.200</td>
<td>14.580</td>
<td>8.100</td>
</tr>
</tbody>
</table>

Table 2.7-2 Propellant Mass

The max lateral amplitude \( \Gamma \) for each load case (20° slosh) is shown in Table 2.7-3.

<table>
<thead>
<tr>
<th>Fuel Load, lb</th>
<th>( \Gamma ), m (LOX)</th>
<th>( \Gamma ), m (LCH4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.355</td>
<td>0.292</td>
</tr>
<tr>
<td>2000</td>
<td>0.453</td>
<td>0.398</td>
</tr>
<tr>
<td>1000</td>
<td>0.482</td>
<td>0.474</td>
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<td>600</td>
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<tr>
<td>400</td>
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<td>0.520</td>
</tr>
<tr>
<td>100</td>
<td>0.573</td>
<td>0.551</td>
</tr>
</tbody>
</table>

Table 2.7-3 Max Lateral Amplitude

Using the information above the damping factor for each load case was estimated using equation (2.7-2) and are presented in table 2.7-4.
Table 2.7-4 Damping Values

<table>
<thead>
<tr>
<th>Fuel Load, lb</th>
<th>γ, LOX</th>
<th>γ, LCH4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.070</td>
<td>0.054</td>
</tr>
<tr>
<td>2000</td>
<td>0.089</td>
<td>0.070</td>
</tr>
<tr>
<td>1000</td>
<td>0.093</td>
<td>0.090</td>
</tr>
<tr>
<td>800</td>
<td>0.108</td>
<td>0.100</td>
</tr>
<tr>
<td>600</td>
<td>0.133</td>
<td>0.125</td>
</tr>
<tr>
<td>400</td>
<td>0.136</td>
<td>0.135</td>
</tr>
<tr>
<td>100</td>
<td>0.140</td>
<td>0.139</td>
</tr>
</tbody>
</table>

As the 4000 lb load case the propellant height decreases, the amount of damping by the baffle goes up indicating decreased damping with deeper submerged baffles. As typical from a design standpoint the vehicle will be required to withstand a minimum damping, and due to the uncertainty in the above predictions, a value of 0.05 will be used for the analysis for conservatism. This should be further verified by experiment.

2.8 Damping Model

With this information, the goal was to modify the analysis model such that approximately 5% damping would occur. For the pendulum model, the magnitude of damping was controlled within the spherical joint constraint within SimWise 4D. The parameters within the spherical joint that controlled losses were the rotational friction coefficient, and effective radius.

By default, no rotational friction is present within the joint constraint. For the present case, the effective radius was input as the pendulum length. The rotational friction
coefficient was found by trial and error. A separate CAD file and SimWise 4D file was used specifically to find the frictional coefficient.

The LOx solid body was placed at a 20 degree angle about the Z axis with respect to the vertical (gravity) and then allowed to “drop”, causing the bodies to swing back and forth around the spherical joint hinge point as shown in figure 2.8-1. One of the bodies remained frictionless (for visual comparison), while a rotational friction coefficient was

Figure 2.8-1 LOx Damping Model About Spherical Joint
input to the second body’s spherical joint constraint. Beginning with a rotational friction coefficient of 0.05, the values were gradually decreased until a damping magnitude of approximately 5% percent was achieved. The magnitude of the amplitude was measured as the angular distance from the neutral position. As shown below, the body starts at an angle of 20 degrees from the neutral (or bottom dead center) position. When damping was added, the magnitude decreased each cycle in a linear manner.

Figure 2.8-2 Friction Coefficient of 0.05, Overdamped
Figure 2.8-3 Friction Coefficient of 0.005, 5.8% Damping

Figure 2.8-4 Friction Coefficient of 0.0045, 5.4% Damping

Figure 2.8-5 Friction Coefficient of 0.0042, 5.0% Damping
CHAPTER 3

ANALYSIS RESULTS

3.1 SIMWISE 4D SIMULATION

The following assumptions were made for this analysis:

- Slosh mass consists of entire propellant mass (conservative)
- Slosh force acts at the fuel center of mass location
- Pendulum slosh mode at theoretical natural frequency – 1st mode only
- Rigid Body Lander
- Damping of 5%

<table>
<thead>
<tr>
<th>Total Propellant Loading, lbs</th>
<th>LOX Start Position, deg</th>
<th>LCH4 Start Position, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation About Z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>+/- 20 (Z)</td>
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<tr>
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<td>+/- 20 (Z)</td>
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<td>800</td>
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<td>600</td>
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<tr>
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<td>Rotation About X</td>
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<tr>
<td>100</td>
<td>+/- 20 (Z)</td>
<td>+/- 20 (Y)</td>
</tr>
</tbody>
</table>

Table 3.1-1 Test Matrix
To analyze the effect of propellant slosh on the rigid body motion of the lander, the lander was run at each load case. The test matrix is shown in table 3.1-1. The orientation, angular velocity, and angular acceleration about each axis was plotted against the propellant level.

3.2 INDUCED ROTATION ABOUT Z AXIS

Figure 3.2-1 All Propellant Bodies Rotated 20 deg About +Z

The first set of analysis cases consisted of all propellant bodies rotated at an initial +20 deg about the +Z axis. The goal was to generate the maximum amount of rotation about the Z axis at each propellant level shown in table 3.1-1. The results are shown below.
Figure 3.2-2 Lander Angular Orientation vs. time, 4000 lb Load Case

Figure 3.2-3 Lander Angular Velocity, 4000 lb Load Case
Figure 3.2-4 Lander Angular Acceleration, 4000 lb Load Case

Figure 3.2-5 Lander Orientation, 2000 lb Load Case
Figure 3.2-6 Lander Angular Velocity, 2000 lb Load Case

Figure 3.2-7 Lander Angular Acceleration, 2000 lb Load Case
Figure 3.2-8 Lander Orientation, 1000 lb Load Case

Figure 3.2-9 Lander Angular Velocity, 1000 lb Load Case
Figure 3.2-10 Lander Angular Acceleration, 1000 lb Load Case

Figure 3.2-11 Lander Orientation, 800 lb Load Case
Figure 3.2-12 Lander Angular Velocity, 800 lb Load Case

Figure 3.2-13 Lander Angular Acceleration, 800 lb Load Case
Figure 3.2-14 Lander Orientation, 600 lb Load Case

Figure 3.2-15 Lander Angular Velocity, 600 lb Load Case
Figure 3.2-16 Lander Angular Acceleration, 600 lb Load Case

Figure 3.2-17 Lander Orientation, 400 lb Load Case
Figure 3.2-18 Lander Angular Velocity, 400 lb Load Case

Figure 3.2-19 Lander Angular Acceleration, 400 lb Load Case
Figure 3.2-20 Lander Orientation, 100 lb Load Case

Figure 3.2-21 Lander Angular Velocity 100 lb Load Case
Figure 3.2-22 Lander Angular Acceleration, 100 lb Load Case

Figure 3.2-23 Average Lander Angular Velocity (Z) vs. Propellant Loading
Figure 3.2-24 Max Lander Angular Velocity (Z) vs. Propellant Loading

Figure 3.2-25 Average Lander Angular Acceleration (Z) vs. Propellant Loading
3.3 INDUCED ROTATION ABOUT Y-AXIS

Figure 3.2-26 Max Lander Angular Acceleration (Z) vs. Propellant Loading

Figure 3.3-1 All Propellant Bodies Rotated +20 deg about + Y axis
Figure 3.3-2 Lander Orientation vs. Time, 4000 lb Load Case

Figure 3.3-3 Lander Angular Velocity vs. Time, 4000 lb Load Case
Figure 3.3-4 Lander Angular Acceleration vs. Time, 4000 lb Load Case

Figure 3.3-5 Lander Orientation vs. Time, 2000 lb Load Case
Figure 3.3-6 Lander Angular Velocity vs. Time, 2000 lb Load Case

Figure 3.3-7 Lander Angular Acceleration vs. Time, 2000 lb Load Case
Figure 3.3-8 Lander Orientation vs. Time, 1000 lb Load Case

Figure 3.3-9 Lander Angular Velocity vs. Time, 1000 lb Load Case
Figure 3.3-10 Lander Angular Acceleration vs. Time, 1000 lb Load Case

Figure 3.3-11 Lander Orientation vs. Time, 800 lb Load Case
Figure 3.3-12 Lander Angular Velocity vs. Time, 800 lb Load Case

Figure 3.3-13 Lander Angular Acceleration vs. Time, 800 lb Load Case
Figure 3.3-14 Lander Orientation vs. Time, 600 lb Load Case

Figure 3.3-15 Lander Angular Velocity vs. Time, 600 lb Load Case
Figure 3.3-16 Lander Angular Acceleration vs. Time, 600 lb Load Case

Figure 3.3-17 Lander Orientation vs. Time, 400 lb Load Case
Figure 3.3-18 Lander Angular Velocity vs. Time, 400 lb Load Case

Figure 3.3-19 Lander Angular Acceleration vs. Time, 400 lb Load Case
Figure 3.3-20 Lander Orientation vs. Time, 100 lb Load Case

Figure 3.3-21 Lander Angular Velocity vs. Time, 100 lb Load Case
Figure 3.3-22 Lander Angular Acceleration vs. Time, 100 lb Load Case

Figure 3.3-23 Average Angular Velocity about +Y vs. Propellant Loading
Figure 3.3-24 Maximum Angular Velocity (Y) vs. Propellant Loading

Figure 3.3-25 Average Angular Acceleration (Y) vs. Propellant Loading
3.4 INDUCED ROTATION ABOUT X-AXIS

Figure 3.4-1 LOX Rotated +/-20 deg (Z). LCH₄ Rotated +/- 20 deg (Y)
Figure 3.4-2 Lander Angular Orientation, 4000 lb Load Case

Figure 3.4-3 Lander Angular Velocity, 4000 lb Load Case
Figure 3.4-4 Lander Angular Acceleration, 4000 lb Load Case

Figure 3.4-5 Lander Angular Orientation, 2000 lb Load Case
Figure 3.4-6 Lander Angular Velocity, 2000 lb Load Case

Figure 3.4-7 Lander Angular Acceleration, 2000 lb Load Case
Figure 3.4-8 Lander Angular Orientation, 1000 lb Load Case

Figure 3.4-9 Lander Angular Velocity, 1000 lb Load Case
Figure 3.4-10 Lander Angular Acceleration, 1000 lb Load Case

Figure 3.4-11 Lander Angular Orientation, 800 lb Load Case
Figure 3.4-12 Lander Angular Velocity, 800 lb Load Case

Figure 3.4-13 Lander Angular Acceleration, 800 lb Load Case
Figure 3.4-14 Lander Angular Orientation, 600 lb Load Case

Figure 3.4-15 Lander Angular Velocity, 600 lb Load Case
Figure 3.4-16 Lander Angular Acceleration, 600 lb Load Case

Figure 3.4-17 Lander Angular Orientation, 400 lb Load Case
Figure 3.4-18 Lander Angular Velocity, 400 lb Load Case

Figure 3.4-19 Lander Angular Acceleration, 400 lb Load Case
Figure 3.4-20 Lander Angular Orientation, 100 lb Load Case

Figure 3.4-21 Lander Angular Velocity, 100 lb Load Case
Figure 3.4-22 Lander Angular Acceleration, 100 lb Load Case

Figure 3.4-23 Average Angular Velocity (X) vs. Propellant Loading
Figure 3.4-24 Max Lander Angular Velocity (X) vs. Propellant Loading

Figure 3.4-25 Average Angular Acceleration (X) vs. Propellant Loading
Figure 3.4-26 Maximum Angular Acceleration (X) vs. Propellant Loading
CHAPTER 4
CONCLUSIONS

As seen in the figures within sections 3.2 through 3.4, the magnitude of the induced motion on the lander created by the fuel slosh decreases with decreasing propellant load starting at the 2000 lb load case. The induced motion has a smaller magnitude at the higher 4000 lb loading due to the increased slosh frequency at higher fuel heights. Smaller free surface area correlates to smaller slosh forces. In the spherical tanks the max slosh occurs near the mid-fill level of the tank where the free surface is at a maximum, as decreases as the tank moves towards being full or empty.

The Morpheus lander studied in this paper will exhibit up to 25 deg/s of initial velocity about the x-axis, 23.5 deg/s about the y-axis, and 26.5 deg/s about the z-axis with 20 deg of slosh at the 2000 lb load level. The above analysis can be redone using any obtained damping ratio or slosh amplitude. Actual induced velocities are expected to be lower as the slosh mass may be reduced from the full propellant load used in this analysis. Lower slosh amplitudes will result in less induced velocity on the lander. The baffles play an important role in dampening out the initial high velocities created by the fuel slosh as seen in the above figures. Baffles should be placed at the location of max slosh force, as damping increases when the baffle is near the free surface. The magnitudes of the disturbances predicted by the simulation should be compared with subscale experiments simulating the specific design configuration.
References


