STATIC VAR COMPENSATOR (SVC) CONTROLLER IMPLEMENTATION

A Project

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Ever Lopez

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STATIC VAR COMPENSATOR (SVC) CONTROLLER IMPLEMENTATION

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by

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Abstract

of

STATIC VAR COMPENSATOR (SVC) CONTROLLER IMPLEMENTATION

by

Ever Lopez

Statement of Problem

As the power grid becomes more interconnected, building new power system infrastructure such as power plants and transmission lines become more difficult due to regulation, financial and numerous other constraints. Methods of maximizing the power flow with reduced losses as well as enhancing the stability of the system have to be analyzed. Implementation of SVCs will allow for optimizing the power system.

Sources of Data

The data was gathered through numerous publications as well as computer aided simulations that served to realize this study.

Conclusions Reached

The system reaches a stabilized state in a much faster time when the system utilizes an SVC controller. Through partitioning of the rotor angle phase plane, analysis for
piecewise linearization, feedback linearization, controllability, observability and pole assignment, the implementation of the SVC is realized.

_______________________, Committee Chair
Dr. Fethi Belkhouche

_______________________
Date
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CHAPTER 1: INTRODUCTION

1.1 Overview

As the power grid becomes more interconnected, building new power system infrastructure such as power plants and transmission lines become more difficult due to regulation, financial and numerous other constraints [1],[2]. The injection of renewable energy sources into the grid must be mentioned due to the fact that interfacing sources like wind and solar to the “traditional” grid pose challenges. Methods of maximizing the power flow with reduced losses as well as enhancing the stability of the system have to be analyzed. There are several ways to maximize power flow and strengthen the stabilizing characteristics of a power network; one example is the use of Flexible AC Transmission System (FACTS) controllers [3],[10].

One of the first FACTS controller brought online in 1974 in a Nebraska power system was designed by Narain G. Hingorani [2]. FACTS controllers utilize solid-state semiconductor devices in order to maximize power flow in the power network. Controllers that use power electronics have been grouped into the FACTS family.

FACTS controllers can be into three subsets: series connected, shunt connected and combined shunt-series connected controllers [3]. The series connected FACTS controllers include Static Synchronous Series Compensator (SSSC), Thyristor Controlled Series Capacitor (TCSC), Thyristor Controlled Series Reactor (TCSR), Thyristor Switched Series Capacitor (TSSC), Thyristor Switched Series Reactor (TSSC) [1]. Among the shunt connected controllers are Static Synchronous Compensator
(STATCOM), Static Condenser (STATCON), Superconducting Magnetic Energy Storage (SMES), Thyristor Controlled Braking Resistor (TCBR), Thyristor Controlled Reactor (TCR), Thyristor Switched Capacitor (TSC), Thyristor Switched Reactor (TSR) and Static Var Compensator (SVC) [1]. The combined shunt-series controllers encompass the Interphase Power Controller (IPC), Unified Power Flow Controller (UPFC) and Thyristor Controlled Phase Shifting Transformer (TCPST) [1].

1.2 Benefits of SVC

In this study, the SVC impact on the power system will be investigated. SVCs affect the active power transfer indirectly by way of voltage control. Indirectly controlling the real power infers voltage regulation of the line voltage by connecting variable reactors and variable capacitors in parallel with the transmission line [1],[2],[3]. The shunt-connected capacitor banks will raise the line voltage when it is below a certain threshold by controlling the reactive power injected into the power system and the shunt-connected reactor bank will lower the line voltage when it is above a certain level by controlling the amount of reactive power extracted from the power system [1],[6]. While the direct method of transmission line voltage regulation is implemented by adding a compensating voltage vectorially to the phase to neutral voltage of the transmission line at the compensation point. A Thyristor Controlled Reactor (TCR) and Thyristor Switched Capacitor (TSC) type SVC is shown in Figure 1.
A brief description of SVCs and their principles is given here. The detailed impact of the SVCs on the power system will be discussed in later chapters. It was shown in [7] that the SVC will change the steady state, the small signal as well as the transient characteristic of the power system. In order to better visualize this, we will look at a two bus system with the SVC compensation point being in the middle of the transmission line with voltage $V_m$. The sending end voltage, $V_1$, and the receiving end voltage is $V_2$. Figure 2 shows a two bus system. The maximum power transfer possible, from bus 1 to bus 2 without the SVC, is given by equation (1.1). With the SVC, the maximum power transferred from bus

$$P_{2\text{max}} = \frac{V_1 V_2}{X}$$  \hspace{1cm} (1.1)
Figure 2 Typical two bus system with SVC connected at the middle point of the transmission line.

The SVC effect on the power system is easily apparent by inspecting equations (1.1) and (1.3). From equation (1.1) the power transferred to bus 2 is half of the power transferred from equation (1.3) if the voltage at the middle of the transmission line, $V_m$, is maintained as close as possible to the sending end voltage, $V_1$. That is, the voltage at the SVC compensating point is regulated such that the regulated voltage and the sending end voltage are almost identical. In order to visualize the damping effect of the SVC on the
power system, the swing equation is taken into account and is defined by equation (1.4). A more detailed study of the swing equation will be presented in later chapters. For now, it will be briefly defined as the primary equation used in learning the stability behavior of a synchronous machine, which dictates the rotational dynamics of that synchronous machine \[6\],[7],[8]. When the second-order swing equation is solved, a relationship between the rotor position (\(\delta\)) and time (\(t\)) is obtained. The swing equation is given by

\[
M \frac{d^2 \delta}{dt^2} = P_{\text{mech}} - P_{\text{elec}}
\]  

(1.4)

where \(P_{\text{mech}}\) is the mechanical power input which is not affected by a small-signal disturbance; \(P_{\text{elec}}\) is the electrical power output; \(\delta\) is the rotor angle of the electrical machine and \(M\) is the inertia constant. Since it was stated that the \(P_{\text{mech}}\) is constant in response to a small-signal disturbance, the only factor that can impact \(\delta\) is the deviation of \(P_{\text{elec}}\) (\(\Delta P_{\text{elec}}\)) which is given as follows,

\[
\Delta P_{\text{elec}} = \frac{\partial P_{\text{elec}}}{\partial V_1} \Delta V_1 + \frac{\partial P_{\text{elec}}}{\partial V_m} \Delta V_m + \frac{\partial P_{\text{elec}}}{\partial \delta} \Delta \delta
\]  

(1.5)

Since it is common practice to place a fairly quick actuating voltage regulator at the generator bus (bus 1) \[7\] and the SVC maintains the middle of the transmission line point voltage (\(V_m\)) at a constant level, the terms \(\Delta V_1\) and \(\Delta V_m\) are equal to zero respectively. Which results in the only way power oscillations are damped, the middle of the transmission line point voltage (\(V_m\)) has to be spanned as a function dependent on the rate of change of the rotor angle with respect to time, which is defined by,
\[ \Delta V_m = K \frac{d \delta}{dt} \]  \hspace{1cm} (1.6)

where \( K \) is the SVC feedback control constant. Now using the closed loop control equation of (1.6), the dynamics of the rotor angle are described by the following equation,

\[ M \frac{d^2 \delta}{dt^2} + \frac{\partial P_{\text{elec}}}{\partial V_m} \left( K \frac{d \delta}{dt} \right) + \frac{\partial P_{\text{elec}}}{\partial \delta} = 0 \]  \hspace{1cm} (1.7)

Given the second order equation defined by (1.7), the characteristic equation is defined by,

\[ S^2 + 2\zeta S + \omega_n^2 = 0 \]  \hspace{1cm} (1.8)

where the damping ratio (\( \zeta \)) and the frequency of oscillation of the power system (natural frequency) are described respectively as follows [5],

\[ \zeta = \frac{1}{2\pi} \frac{\text{Natural Period (seconds)}}{\text{Exponential Time Constant}} \]  \hspace{1cm} (1.9)

and

\[ \omega_n = \sqrt{\frac{1}{M} \frac{\partial P_{\text{elec}}}{\partial \delta}} \]  \hspace{1cm} (1.10)
Since it was stated that the mechanical power ($P_{\text{mech}}$) did not deviate from its normal operating value, the rate of change of the rotor with respect to time, using equation (1.5), can be defined in terms of the active power being transmitted,

$$\frac{d\delta}{dt} \rightarrow \int \Delta P_{\text{elec}} \, dt \quad (1.11)$$

The transient stabilization impact on the power system by the SVC can quickly be visualized by noting that the reactive shunt compensation can greatly increase the maximum amount of power that can be transmitted and assuming adequate timing of the controls, the SVC will modify the power flow of the power system at times of disturbances and post disturbances [6]. Thus, the transient stability limit will be larger and robust power oscillation damping will be achieved. The transient stability enhancement of the power network can be described effectively by way of the equal area criterion [6],[7],[8],[9].

A detailed discussion will be presented in the following chapters of the linearization technique used to describe the non-linearity of a multi-machine network. The linear models of the multiple generator power system will be used in the optimization of SVCs to enhance the various criteria for stability.
CHAPTER 2: DYNAMICS OF POWER SYSTEM
WITH NO SVC COMPENSATION

In this chapter, we study the steady-state stability of an individual generator in where no SVC compensation is included. In order to facilitate the study, the following assumptions are made [7]:

1) The normal operation mechanical input power does not deviate during the transient interval.
2) Unbalanced power or damping is not considered.
3) The generator can be graphically represented by a fixed voltage, which is followed by a transient reactance, as shown in figure 3.
4) The generator rotor mechanical angle matches the electrical phase angle of the fixed voltage followed by the transient reactance.
5) In the case that a load is connected to the terminal bus of the generator, the load can be represented by a fixed impedance/admittance to neutral.

![Figure 3 Fixed generator voltage followed by a transient reactance](image_url)
2.1 System with no Damping

Given the assumptions, the swing equation with no damping constraint, is given by

\[
\dot{\omega} = \delta
\]

\[
\ddot{\omega} = M_1^{-1} P_i - b_1 M_1^{-1} \sin(\delta_i)
\]  

(2.1)

Equation (2.1) is written in a more general form that can be used for a multimachine system. For a single machine

\[
\frac{2H_1}{\omega_0} \ddot{\delta}_i = P_m - P_e
\]

(2.2)

where \(P_m\) and \(P_e\) are the mechanical and electrical power respectively. The electrical power \(P_e\) is given by

\[
P_e = \frac{EV}{X} \sin(\delta_i)
\]

(2.3)

where the maximum power transfer between nodes E and V is

\[
P_{\text{max}} = \frac{EV}{X}
\]

(2.4)

and \(P_m\) is a constant as stated previously.
A disturbance ($\delta_\Delta$) is now introduced, in order to characterize the system when a deviation from a steady-state point ($\delta_0$) occurs. The rotor angle ($\delta_{11}$) and the electrical power ($P_{e1}$) respectively are expressed by

$$\delta_{11} = \delta_0 + \delta_\Delta$$

(2.5)

$$P_{e1} = P_{e0} + P_{e\Delta}$$

(2.6)

where $P_{e0}$ is the steady-state operation electrical power and $P_{e\Delta}$ is the deviation of electrical power from steady-state operation. The swing equation is now be given by

$$\frac{2H}{\omega} \dddot{\delta}_\Delta = P_m - (P_{e0} - P_{e\Delta})$$

(2.7)

Since $P_m$ and $P_{e0}$ are constant and do not contribute to the disturbance, (2.7) can be reduced to

$$\frac{2H}{\omega_0} \dddot{\delta}_\Delta = -[P_{\max} \cos(\delta_0)]\delta_\Delta$$

(2.8)

The term $-[P_{\max} \cos(\delta_0)]$, is defined as the synchronizing power coefficient ($P_{s-p}$). The synchronizing power coefficient identifies the rate of change of the power angle curve at the steady-state operating point ($\delta_0$). A closer look at equation (2.8) in terms of the synchronizing power coefficient can lead to some observations as follows

$$\frac{2H}{\omega_0} \dddot{\delta}_\Delta = -[P_{s-p}]\delta_\Delta$$

(2.9)
The swing equation for incremental rotor angle (2.9):

1) Is a second order differential equation (ODE)
2) The rotor angle disturbance term is the variable of interest
3) Is a linear equation
4) The behavior of the rotor angle disturbance term depends on the synchronizing power coefficient

Given equation (2.9) the characteristic equation is given by

\[ S^2 + \frac{\omega_o \mid P_{s-p} \mid}{2H} = 0 \]  (2.10)

and

\[ S^2 = -\frac{\omega_o \mid P_{s-p} \mid}{2H} \]  (2.11)

From the characteristic equation it can be shown that the generator rotor angle operates in the range of 0 to \( \pi/2 \) in order for the equilibrium (steady-state) point to maintain stability, which relates to the synchronizing power coefficient being positive for stable operation (bounded disturbance) and being negative for unstable operation (increasing disturbance with time).

Now equation (2.1) in state-space form is used for the first generator in order to simulate the swing equation with no damping and is provided by
with \( b_1 = 950 \text{ kW} \), \( M_1^{-1} = 4.8 \times 10^{-6} \) and \( P_1 = 475 \text{ kW} \) (mechanical input power from the turbine and the governor). As can be seen from figure 4, the oscillations are sustained with time when there is no damping present. The sustained oscillation behavior can also be predicted by looking at the roots of equation (2.11) and noticing that they are located on the imaginary axis of the s-plane.

\[
\begin{align*}
\begin{bmatrix}
\dot{\delta}_1 \\
\dot{\omega}_1
\end{bmatrix} &=
\begin{bmatrix}
0 & 1 \\
-4.58 \cos(\delta_1) & 0
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\omega_1
\end{bmatrix}
\end{align*}
\]  
(2.12)

Figure 4 Simulation of the swing equation with no damping component
2.2 System with Damping

The damping effects, on the swing equation are now considered. In this case, the swing equation is described by [4]

\[
\begin{align*}
\dot{\omega}_i &= \ddot{\delta}_i \\
\dot{\omega}_i &= -D_i J_i^{-1} \omega_i + M_i^{-1} P_i - b_i M_i^{-1} \sin(\delta_i)
\end{align*}
\]  

(2.13)

In the presence of a disturbance, equation (2.4) is expressed as

\[
\begin{align*}
\frac{d\ddot{\delta}_\lambda}{dt} &= \omega_\lambda \\
\frac{d^2\ddot{\delta}_\lambda}{dt^2} + D_i J_i^{-1} \frac{d\ddot{\delta}_\lambda}{dt} + b_i M_i^{-1} \cos(\delta_0) \ddot{\delta}_\lambda &= 0
\end{align*}
\]  

(2.14)

As was the case for the non-damping scenario, equation (2.14) is a second order linear ordinary differential equation. The characteristic equation is given by

\[
S^2 + D_i J_i^{-1} S + b_i M_i^{-1} \cos(\delta_0) = 0
\]  

(2.15)

The roots of equation (2.15) are

\[
S_{1,2} = \frac{-D_i J_i^{-1} \pm \sqrt{(D_i J_i^{-1})^2 - 4(b_i M_i^{-1} \cos(\delta_0))}}{2}
\]  

(2.16)

The discriminant is negative in most cases which results in complex solutions. The system response is oscillatory such that the angular frequency of oscillation is identical to the case where no damping occurs and the system is stable for \(D_1\) and \(b_1 M_1^{-1} \cos(\delta_0)\) greater than zero. The system is unstable if one of these terms is negative [5]. In order to simulate the system with damping, equation (2.13) is used in the state-space form and is described by
\[
\begin{bmatrix}
\dot{\delta}_1 \\
\dot{\omega}_1
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-b_i M_i^{-1} \cos(\delta_i) & -DJ_i^{-1}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\omega_1
\end{bmatrix}
\] (2.17)

Using a value for the damping constant \((D_1)\) of 95, the state-space representation is given as follows,

\[
\begin{bmatrix}
\dot{\delta}_1 \\
\dot{\omega}_1
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-4.58 \cos(\delta_1) & -0.173
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\omega_1
\end{bmatrix}
\] (2.18)

In an effort to provide the reader with clear linearization techniques, the same analysis will be executed by calculating the steady-state rotor operating point \((\delta_0)\) and using the Jacobian matrix to solve the system of differential equations. We begin by using equation (2.13), slightly modified, and using the fact that angular velocity \((\omega_1)\) is zero at steady-state. The system of equations is given by

\[
\begin{align*}
\dot{\delta}_{A1} &= \omega_{A1} \\
\dot{\omega}_{A1} &= \frac{-D_i}{J_i} \omega_{A1} - b_i M_i^{-1} \cos(\delta_0) \delta_{A1} + M^{-1}(P_m + P_\Delta)
\end{align*}
\] (2.19)

and the system at steady-state operating condition \((\delta_0)\) is given by

\[
\dot{\omega}_1 = -b_i M_i^{-1} \sin(\delta_0) + M^{-1}(P_m) = 0
\] (2.20)

Now the steady-state operating can be determined by

\[
\delta_0 = \text{Arc} \sin\left(\frac{P_m}{b_i}\right)
\] (2.21)
where $b_1$ is equal to 950kW as previously stated, $P_m$ is 475kW and steady-state operating rotor angle ($\delta_0$) is calculated to be 30.0 degrees. We now use $\delta_0$ to determine the Jacobian matrix ($J^{(jm)}$) where we have functions $F_1(\delta_1, \omega_1)$ and $F_2(\delta_1, \omega_1)$, thus the Jacobian is generally given by

$$J^{(jm)} = \begin{bmatrix} \frac{\partial F_1}{\partial \delta_1} & \frac{\partial F_1}{\partial \omega_1} \\ \frac{\partial F_2}{\partial \delta_1} & \frac{\partial F_2}{\partial \omega_1} \end{bmatrix}$$

(2.22)

For our case, the Jacobian is as follows

$$J^{(jm)} = \begin{bmatrix} 0 & 1 \\ -b_1 M_1^{-1} \cos(\delta_0) & - \frac{D_1}{J_1} \end{bmatrix}$$

(2.23)

The state-space linearized swing equation is now given by

$$\begin{bmatrix} \dot{\delta}_{\Delta 1} \\ \dot{\omega}_{\Delta 1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b_1 M_1^{-1} \cos(\delta_0) & - \frac{D_1}{J_1} \end{bmatrix} \begin{bmatrix} \delta_{\Delta 1} \\ \omega_{\Delta 1} \end{bmatrix}$$

(2.24)

Substituting all the given values, equation (2.24) reduces to

$$\begin{bmatrix} \dot{\delta}_{\Delta 1} \\ \dot{\omega}_{\Delta 1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3.96 & -0.173 \end{bmatrix} \begin{bmatrix} \delta_{\Delta 1} \\ \omega_{\Delta 1} \end{bmatrix}$$

(2.25)

As can be seen in figure 5, the oscillations dissipate with time for both the linear and the non-linear swing equation. The solution converges to the stable equilibrium point. The damping coefficient is a result of a difference in angular velocity between the air gap
field and the rotor field, which results in the creation of an induction torque on the rotor that will decrease the difference in the respective velocities [6].

**Figure 5** Simulation of swing equation in non-linear form and linear form
CHAPTER 3: DYNAMICS OF POWER SYSTEM
WITH SVC COMPENSATION

3.1 Basic SVC Network

Power system dynamics are now studied with the addition of SVC compensation. The SVC is shunt connected to the power network. There are studies for optimal placement of an SVC [1]; in this study, the SVC is located as shown in figure 6.

![Diagram of Power System with SVC compensation and multi-machine network](image)

Figure 6  Power System with SVC compensation and multi-machine network

The power network consists of a multi-machine system, a single generator (gen), which will be compensated by the SVC, mechanical power input \( P_m \) to the generator from the turbine and governor, SVC (variable susceptance \( X_{SVC} \)), feedback control to the SVC \( K \). Buses M, G, and C are shown in figure 6. The generator \( x_g \) and transmission line
(xl) equivalent reactance and the transmission network that connects the single generator to the multi-machine network are also shown in the figure.

The swing equation with SVC compensation can be expressed by the following system [2]

\[
\begin{align*}
\dot{\delta}_M &= \omega_M \\
\dot{\omega}_M &= \frac{D}{J} \omega_M - \frac{b}{xM} \sin(\delta) \delta_M + \frac{P_m + P_\Delta}{M_1}
\end{align*}
\]

(3.1)

where \(x\) and \(b\) are defined by

\[
x = x_g + x_t - X_{\text{svc}} x_g x_t
\]

(3.2)

\[
b = E_c E_g
\]

(3.3)

\(E_c\) and \(E_g\) are simulated voltages (virtual voltages) on bus C and G respectively, as opposed to real voltage measurements on the same buses. As was stated in the previous chapter, \(P_\Delta\), is the power disturbance on the single generator rotor caused by the multi-machine network.

Since the mechanical power (\(P_m\)) is the input to the single generator system, it is stated that the output to the system will be the rotor angle (\(\delta\)) in an effort to present a clear explanation of the system. The state-space expression for equation (3.1) is given by [4]

\[
\begin{align*}
\dot{\psi} &= A\psi + B_1 x^{-1} + B_2 (P_m + P_\Delta) \\
y &= C\psi
\end{align*}
\]

(3.4)

where \(\psi\), \(A\), \(B_1\), \(B_2\), and \(C\) are defined respectively as follows,
It should be noted that $B_1$ is a function of $\delta_{\Delta_1}$.

\[
B_1 = \begin{bmatrix} 0 & -b \sin(\delta_{\Delta_1}) \\
                        1 & M_1 \end{bmatrix}
\] (3.7)

It is clear that $C$ represents the output matrix. Given the non-linearity of equation (3.4) a linearization method has to be implemented to solve the system of differential equations.

### 3.2 Linearization

The linearization process will begin by defining the rotor angle ($\delta$) phase plane broken up into numerous sections encompassed by its width (w) and expressed as $\delta_i$, where “i” is the section 1, 2, 3…, n. The width of each section has to be small enough such that the domain for each partition is defined by [4]

\[
\text{Domain} : (\delta_i \leq \delta < \delta_{i+1})
\]
and it can be stated that an approximation of

\[ \delta \approx \delta_i \] (3.10)

is fairly accurate and it is certain for every \( \delta \) in the domain of interest. A new state-space representation of the power system is defined noting that it is a piecewise linear approximation of equation (3.4) and is given by the following [4],[11],[12]

\[
\begin{align*}
\dot{\psi}_{\Delta i} &= A\psi_{\Delta i} + B_1x^{-1} + B_2(P_m + P_\Lambda) \\
y &= C\psi_{\Delta i}
\end{align*}
\] (3.11)

where the state vector of the system \( (\psi_i) \) and \( B_1 \) are respectively defined as

\[
\psi_i = \begin{bmatrix} \delta_{\Delta i} \\ \omega_{\Delta i} \end{bmatrix}
\] (3.12)

\[
B_1 = \begin{bmatrix} 0 \\ -b\sin(\delta_{\Delta i}) \\ M \end{bmatrix} = \begin{bmatrix} 0 \\ -\Gamma_i \end{bmatrix}
\] (3.13)

\( B_1 \) is a function of \( \delta_i \) and is a constant value for every different \( I \) since \( \delta_i \) is constant in each interval. \( A, B_2, C \) are defined in equations (3.6), (3.8) and (3.9) respectively. Since \( \psi(t) \) and \( \psi_{\Delta i}(t) \) are in the domain of interest, the initial time constraint parameters are also in the domain of interest which leads to correlation that the dynamics of equation (3.4) can be approximated by equation (3.11) [4].

The same reasoning used for in the previous chapter for utilizing the Jacobian is implemented here as well. Using the system of equations given in equation (3.1), the Jacobian is given by
In order to study the pole placement technique for SVC design of the power system, we first must study the controllability of the network and the observability as well.

### 3.3 Controllability

Given that an nth-order feedback control system has a characteristic equation of the following form [5]

\[
S^n + z_{n-1}S^{n-1} + \ldots + z_1S + z_0 = 0
\]

and is the closed loop polynomial as well. The constants associated with the s-terms except the highest power s-term (because it has coefficient of one) dictate the networks closed loop pole orientation and there are a total of \( n \) constant coefficients. Therefore, \( n \) parameters can be varied such that they are connected to the \( n \) coefficients and results in the ability to place the closed loop poles in any chosen location, which will be used for stabilization purposes. It can be stated, that the system is characterized at any instant by the state of the network, which is a pool of all the state variables and their corresponding values. A system that is totally controlled can describe the ability of an external input to modify the state of the network from any initial condition point to any other resting state.
in a certain amount of time. The controllability expression of equation (3.11) in terms of \( x^{-1} \) is given by [4]

\[
T_{\text{control},i}^x = [sI - A | B_i(\delta_i)] = \begin{bmatrix}
  s & -1 & 0 \\
  0 & s + \frac{D}{J} & -\frac{b\sin(\delta_i)}{M}
\end{bmatrix}
\]

(3.16)

In order to verify that equation (3.15) is controllable, the greatest number of linearly independent columns or greatest number of linearly independent rows have to be determined which define the rank of \( T_{\text{control},i}^x \) [5]. It should be noted that linearly independent columns (column rank) and linearly independent rows (row rank) are always equal. The rank of equation (3.15) is 2. Therefore it can be stated that the system is controllable in terms of \( x \) when \( \sin(\delta_i) \) is not equal to zero and the poles are assignable through the state-space. \( \sin(\delta_i) \) equal to zero can easily be avoided by specifically defining \( \delta_i \) [4].

A controllability expression in terms of the mechanical input power, \( P_m \), and the corresponding rank has to be determined as well in order classify the multi-machine network as completely controllable. The controllability matrix in terms of \( P_m \) is given as follows [4]

\[
T_{\text{control},i}^{P_m} = [sI - A | B_2] = \begin{bmatrix}
  s & -1 & 0 \\
  0 & s + \frac{D}{J} & M^{-1}
\end{bmatrix}
\]

(3.17)
To determine the controllability of equation (3.16) in terms of the mechanical power, the rank of $T_{\text{control,}i}^{P_m}$ is determined which equals two. Thus, (3.16) is controllable in terms of $P_m$ and the poles can be placed within the state-space of equation (3.16). The observability of the network is now studied.

### 3.4 Observability

The observability of a system dictates whether the state of the system presently can be determined only by the output of the system [5]. Essentially the objective is to determine the behavior of the whole network by analyzing the output of the system. The observability expression in terms of the output of the network which is the rotor angle, is given by [4]

$$O_{\text{observe,}i} = \left[ sI - A \right] = \begin{bmatrix} s & -1 \\ 0 & s + \frac{D}{J} \\ 1 & 0 \end{bmatrix}$$

(3.18)

The rank of equation (3.17) has to be determined in order to verify that the network is observable. The rank of $O_{\text{observe,}i}$ is equal to two. Therefore, the system is observable, as was shown in [4], by way of the Popov-Belevich-Hautus rank criterion, and the initial condition vector at an initial time $t_0$ can be determined from the control vector with the ability of the system to sample the output over a certain amount time from $t_0$. 
3.5 Pole Placement

In continuing to seek the implementation of pole placement in the design of the SVC controller, the closed-loop control for the SVC is given by

\[ x^{-1} = [K_{1i}, K_{2i}] \psi_i = K \psi_i \]  (3.19)

where \( K_{1i} \) is the feedback control assigned to the desired pole 1 and \( K_{2i} \) is the feedback control assigned to the desired pole 2 [4]. It should be noted that the variable susceptance is within the range, \( X_{\text{min SVC}} \leq X_{\text{SVC}} \leq X_{\text{max}} \), and constrained such that \( X_{\text{min SVC}} \) is less than zero as well as \( X_{\text{max}} \) being greater than zero [1],[2],[3],[4]. The state feedback of equation (3.18) is now integrated into the closed-loop state-space equation of (3.11) and is given by

\[ \dot{\psi}_i = (A + B_i[K_{1i}, K_{2i}])\psi_i + B_2(P_m + P_A) \]
\[ y = C\psi_i \]  (3.20)

From equation (3.19) the closed-loop characteristic equation is obtained for each narrow partition (i) and is as follows [4]

\[ |sI - A - B_i[K_{1i}, K_{2i}]| = \begin{vmatrix} s & -1 \\ -K_{1i}\Gamma & s + \frac{D}{J} - K_{2i}\Gamma \end{vmatrix} \]
\[ = s^2 + \left( \frac{D}{J} - K_{2i}\Gamma \right)s - K_{1i}\Gamma \]  (3.21)

The closed-loop poles for equation (3.20), using the quadratic equation, are given by [4]

\[ P_{c1,2} = -\frac{(D/J) - K_{2i}\Gamma_i}{2} \pm \sqrt{\left(\frac{(D/J) - K_{2i}\Gamma_i}{2}\right)^2 + K_{1i}\Gamma_i} \]  (3.22)
It can be clearly seen in equation (3.21) that by dictating the values for the feedback control, \(K_{1i}\) and \(K_{2i}\), the poles can be forced to have real parts that are negative which is a requirement for stability. As was shown in [4],[11] the state feedback controlled SVC of equation (3.18) is valid for design and stabilization of equation (3.19).

The ratio of the output rotor angle (\(\delta\)) and the input mechanical power (\(P_m\)) is determined, which is the transfer function given by [4]

\[
G_{t,e} = C(sI - A - B_1[K_{1i} \ K_{2i}])^{-1}B_2
\]

(3.23)

and it can be further reduced to the following expression

\[
G_{t,e} = \frac{1}{M(s^2 + s\frac{D}{J} - K_{2i}\Gamma_i) - K_{1i}\Gamma_i}
\]

(3.24)

Using the closed-loop poles of equation (3.23) and the desired eigenvalues, \(\lambda_1\) and \(\lambda_2\), of the wanted closed-loop transfer function, SVC design through pole assignment is realized. The wanted closed-loop transfer function with the corresponding eigenvalues is given by

\[
G_t(s) = \frac{1}{[M(s - \lambda_1)(s - \lambda_2)]} = \frac{1}{[M(s^2 - (\lambda_1 + \lambda_2)s + \lambda_1\lambda_2)]}
\]

(3.25)

Equations (3.23) and (3.24) are now set equal to each other in order to relate the desired eigenvalues to the SVC feedback control gains, \(K_{1i}\) and \(K_{2i}\), which results in the following

\[
\frac{1}{[M(s^2 - (\lambda_1 + \lambda_2)s + \lambda_1\lambda_2)]} = \frac{1}{[M(s^2 + s\frac{D}{J} - K_{2i}\Gamma_i) - K_{1i}\Gamma_i]}
\]

(3.26)

Equating coefficients from equation (3.25) results in the following
\[-K_{ii}\Gamma_i = \lambda_i \lambda_2\]  \hspace{1cm} (3.27)

and

\[\frac{D}{J} - K_{ii}\Gamma_i = -\lambda_i + \lambda_2\]  \hspace{1cm} (3.28)

Solving for the closed-loop feedback control gains, \(K_{1i}\) and \(K_{2i}\), respectively gives the following expressions

\[K_{1i} = -\frac{\lambda_i \lambda_2}{\Gamma_i}\]  \hspace{1cm} (3.29)

\[K_{2i} = \Gamma_i^{-1}(\lambda_i + \lambda_2 + \frac{D}{J})\]  \hspace{1cm} (3.30)

It can be seen from equations (3.28) and (3.29) that by dictating the closed-loop poles, the gain is adjusted. Since the condition for stability states that the real part of the eigenvalues has to be negative, they can be chosen such that the system is always stable in all the partitions of the phase plane. As was established, the system is controllable and observable, therefore the resting state of the network will always be forced toward the steady-state operating rotor angle that is desired. As figure 7 shows, the system reaches a stabilization point in a much faster time when SVC compensation is implemented. In order to minimize the steady-state error, the gain \(K_1\), is varied. As can be seen in comparing figure 7 and figure 8, where \(K_1\) was assigned a value of 3 and a value of 1 for the respective figures, the steady-state behavior is adjusted.
Figure 7  System under SVC compensation where $K_1 = 3$

Figure 8  System under SVC compensation where $K_1 = 1$
In order to improve the transient response, the gain $K_{2i}$ is adjusted, which is clearly shown in figure 9.

![Figure 9 System under SVC compensation for $K_{2i} = 5$](image)

### 3.6 Feedback Linearization

The goal of feedback linearization is to use feedback law under which the closed loop system becomes linear [13]. Consider a non-linear system under the following form

\[
\dot{x} = f(x) + g(x)u \\
y = h(x)
\]  

(3.31)

For single-machine systems, we suggest the following feedback linearization law,
\[
x^{-1} = \frac{-K_1 \delta - K_2 \omega + K_3 u}{\delta}
\]

(3.32)

where

\[K = [K_1 \ K_2]\]

(3.33)

is the state feedback gain matrix and \(K_3\) is the feed-forward gain and the \(u\) parameter is a chosen offset value. Utilizing the feedback gain matrix and the feed-forward gain in the swing equation, gives the following equation

\[
\begin{align*}
\dot{\delta} &= \omega \\
\dot{\omega} &= \left[- \frac{D}{J} + \frac{K_3 b}{M} \sin(\delta_0)\right] \omega + \frac{K_1 b \sin(\delta_0)}{M} \delta + \frac{P_m + P_{\lambda}}{M} - \frac{K_3 b \sin(\delta_0)}{M} u
\end{align*}
\]

(3.34)

In order to provide a transparent explanation of feedback linearization, a numerical example is provided, with the previously given values used. The desired eigenvalues \((\lambda_1, \lambda_2)\) and the feed-forward gain \((K_3)\), will be chosen which will allow for the calculation of \(K_1\) and \(K_2\). The swing equation in linear form using feedback linearization is as follows

\[
\begin{align*}
\dot{\delta} &= \omega \\
\dot{\omega} &= (-0.1727 + 2.196K_2) \omega + 2.196K_1 \cdot \delta - 2.196K_3 \cdot u + 2.38
\end{align*}
\]

(3.35)

Now we find a relationship between the state feedback gains \((K_1, K_2)\) and the desired eigenvalues \((\lambda_1, \lambda_2)\) which results in the given matrix equation

\[
\det(\lambda I - A) = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2.196K_1 & -0.1727 + 2.196K_2 \end{bmatrix}
\]

(3.36)

and the resulting expression is determined to be
\[ \det(\lambda I - A) = \lambda^2 - (2.196K_2 - 0.1727)\lambda - 2.196K_1 \] (3.37)

Since the desired characteristics of the eigenvalues are known, such that the system is stable, the solution of the eigenvalues has to be in the following form

\[ (\lambda - \lambda_1)(\lambda - \lambda_2) = \lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2 \] (3.38)

We now set expressions (3.37) and (3.38) equal to each other so that we can correlate the desired eigenvalues and the feedback gains by equating coefficients which results in the following

\[ \lambda_1 + \lambda_2 = 2.196K_2 - 0.1727 \]
\[ \lambda_1\lambda_2 = -2.196K_1 \] (3.39)

and results in expressions for \( K_1 \) and \( K_2 \) respectively,

\[ K_1 = -\frac{\lambda_1\lambda_2}{2.196} \] (3.40)
\[ K_2 = \frac{\lambda_1 + \lambda_2 + 0.1727}{2.196} \] (3.41)

We now choose the desired eigenvalues to be -0.3 for \( \lambda_1 \) and -5 for \( \lambda_2 \) where the corresponding \( K_1 \) and \( K_2 \) are -0.68, -2.33 respectively. By tuning \( K_3 \), it is quickly realized, that it dictates the steady-state behavior and can be used to minimize the steady state error of the system. A \( K_3 \) value was tuned to -1.45 which corresponds to a steady-state error of 0.0055 which is shown in figure 10. In order to verify the impact of \( K_1 \) and
on the system, several values of \( K_1 \) and \( K_2 \) are calculated, for constant \( \lambda_2 \) and varying \( \lambda_1 \), the results are shown in table 1. Figures 11 and 12 clearly show that as \( K_1 \)

\[ \text{Table 1 } K_1 \text{ and } K_2 \text{ values varying } \lambda_1 \text{ and holding } \lambda_2 \text{ constant} \]

and \( K_2 \) vary, the transient response changes. We continue to calculate different values for \( K_1 \) and \( K_2 \) to further investigate the transient response. Table 2 contains values of \( K_1 \) and \( K_2 \) for a constant \( \lambda_1 \) while varying \( \lambda_2 \). Like figures 11 and 12, figures 13 and 14 show

\[ \text{Figure 10 Steady state error minimization for } K_3=-1.45, K_1=-0.68, K_2=-2.3 \]
Figure 11 Transient response for $K_1=-0.22$ and $K_2=-2.2$

Figure 12 Transient response for $K_1=-1.4$ and $K_2=-2$
by varying the values of the feedback gain, the transient response is altered correspondently. Therefore, it can be concluded that in order to minimize the steady-state error, $K_3$ is modified while if one wishes to change the transient behavior of the system, $K_1$ and $K_2$ must be altered.

\[
\begin{array}{cccc}
\lambda_1 & \lambda_2 & K_1 & K_2 \\
-0.3 & -1 & -0.13661 & -0.51334 \\
-0.3 & -2 & -0.27322 & -0.96872 \\
-0.3 & -3 & -0.40984 & -1.42409 \\
-0.3 & -4 & -0.54645 & -1.87946 \\
-0.3 & -5 & -0.68306 & -2.33484 \\
-0.3 & -6 & -0.81967 & -2.79021 \\
\end{array}
\]

Table 2 $K_1$ and $K_2$ values holding $\lambda_1$ constant and varying $\lambda_2$

![Figure 13 Transient response when $K_1=-0.14$ and $K_2=-0.51$](image)
Figure 14 Transient response for $K_1=-0.82$ and $K_2=-2.8$
CHAPTER 4: CONCLUSION

As was studied in this literature, the SVC controller provides many benefits to the power system. From its inception in the 1970’s, the SVC and the whole family of FACTS controllers are used presently in order to maximize power transfer capability as well as the stabilization behavior of the system in an acceptable period of time. As the power system becomes more interconnected, distributed generation becomes more prevalent and transmission infrastructure is more difficult to build, optimization of the current system must be studied as well as utilizing the tools that are currently available such as FACTS controllers.

In this study, we showed how to analyze the system without SVC compensation and with SVC compensation; this was accomplished through the exploitation of various methods used to solve the swing equation. It was demonstrated that the swing equation can be solved by way of computer aided software in its non-linear form or the system can be linearized through piecewise linearization as well as feedback linearization. Although there are many ways to solve the swing equation, it was shown that the SVC does improve the steady-state capabilities and transient behavior of the system by only adjusting several gain parameters. Therefore, it is advantageous to implement an SVC controller in a power system and as was demonstrated, the complexity is minimal.
REFERENCES


