THE SOCIAL MATH LITERACY PROJECT: A PROFESSIONAL DEVELOPMENT
THAT SCAFFOLDS TEACHING OPEN-ENDED MATH PROBLEMS WITH AN
EMPHASIS IN SOCIAL JUSTICE

A Project

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by

Cecilia Lopez-Avaria

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A Project

by

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Date

Graduate and Professional Studies in Education
Abstract

of

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Statement of the Problem

In teaching and learning mathematics, the challenge has always been giving the subject a more realistic approach as well as providing students with opportunities to investigate and discover math concepts. Historically, teachers spend vast amounts of time talking and lecturing students, giving the subject a more banking approach rather than a problem-posing approach. Such results were confirmed by Boaler and Staples’s study (2008) in which they concluded, “48% of the [class] time students were practicing methods in their books, working individually, and students presented work for approximately 0.2% of the time” (p. 619). There has been a lack of a problem-based approach to teaching mathematics, but more importantly, there has been a lack of giving the mathematics the opportunity to shine as a tool for students to become quantitatively literate. The Social Math Literacy Project opens doors to the educational community to explore and become aware of the social challenges in the students’ everyday lives and to grasp these challenges rather than ignore them through mathematics.
Sources of Information

The project was created using H. Freudenthal’s ideas of Realistic Mathematics Education (RME) that focus on teaching mathematics as a human activity and Freire’s problem-posing education as the practice of freedom. I created a professional development that approaches a problem-based mathematics with a social justice emphasis, transforming RME problems into problems that will ensure social discourse in math classes as well as achievement of math skills.

Conclusions Reached

As I researched and learned about the importance of teaching and learning mathematics with a problem-based focus, I also learned about the lack of a more social justice approach to education in general. A problem-centered mathematics curriculum can serve as a means to achieving democratic values that will eventually translate to a more tolerant, peaceful, and empathetic society. In other words, a curriculum, such as the one developed in this project, that respects students’ funds of knowledge, their home cultures, and their intelligences will ensure their success as individuals in a changing, demanding, and dynamic society.

_________________________________________, Committee Chair
Margarita Berta-Ávila, Ed.D.

_________________________________________
Date
DEDICATION

To all stay-at-home mothers seeking reinvention.

To my husband who patiently waited the end of my journey.

To my daughters who understood there was more than mom within myself.

To my family and friends in Chile.
ACKNOWLEDGMENTS

Thank you, Dr. Margarita Berta-Ávila, for your guidance, support, and endless conversations to seek the end of this journey. I could not have completed this project without your knowledge, help, and encouragement. Thank you, Dr. Albert Lozano, for understanding each one of our limitations of us members of the cohort. Thank you to all BMED MA teachers: Dr. Lisa William-White, Dr. José Cintrón, and Dr. Maria Mejorado who contributed with information and ideas. Thank you, Dr. Scott Farrand, for your generous guidance through the mathematics component of the project.
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Chapter 1

INTRODUCTION

All subjects have something to contribute in the promotion of equity and that mathematics, often regarded as the most abstract subject removed from responsibilities of cultural or social awareness, has an important contribution to make. (Boaler & Staples, 2008, p. 639)

Equal access to mathematics education is the right of every student, as the subject provides the tools to make sense and have a better understanding of numerous challenges encountered in life situations, “including activities that explore the cost of war and poverty and the connections between race, privilege and inequity” (De Freitas, 2008, p. 87). A real life application of the subject has been missing and, in 1989, The National Council of Teachers of Mathematics recommended curricula provide opportunities to develop an understanding of mathematical models, structures, and simulations applicable to many other disciplines such as business, economics, linguistics, biology, medicine, and sociology, thus making mathematics more real. Mathematics would then be a “live” subject that can be used beyond solving disengaging problems requiring repetition and recitation of algorithms.

When mathematics is given a real-life approach, it can help people calculate and understand the real cost of a loan, find the best cellular phone plan, pay for college, file for taxes, create awareness of water and food distribution, or have an idea of defense budgets. These examples of the applied use of mathematics help provide meaning to the
subject by presenting the learners the tools to make connections to everyday situations, to give them the opportunity to talk about math, and to use math to analyze society and its norms in a critical manner. That is, through real-life math problems with an emphasis on critical social themes, students can be engaged in situations in which they have to question their roles in society and be able to understand and change situations in which they have been overpowered. According to Gutierrez (2010), these situations can be used to help students “make sense of data in ways that help them see the humanity behind the numbers and to use mathematics as a tool for exposing and analyzing injustices in society and as a means for convincing others of a particular (often non dominant) point of view” (p. 5). In other words, students become mathematically literate to make informed decisions to change situations that will eventually affect their positionality in the world as democratic citizens. Therefore, conveying the subject in a manner that promotes democracy by offering the students themes that create awareness of how numbers are used to promote oppression involves teaching skills that cannot be acquired using the uniform traditional approach to teaching. Consequently, a more just focus on mathematics requires an approach centered on problem posing strategies, as these help make the subject accessible to everyone.

The goal is to promote access to critical thinking in an equitable manner involving the use of methodologies that include the students’ life experiences as well as a strong focus on real-life situations. Hence, an environment in which they empower themselves to become accountable for their education and their future as active participants in society
can be fostered. But why is it important to give mathematics a social justice approach? In *Pedagogy of the Oppressed* (Freire, 1993), Shaull argued that every human being no matter how “ignorant” or submerged in the “culture of silence” (p. 32) is capable of looking critically at the world in a dialogical encounter with others. Therefore, this argument becomes the new role of the subject as mathematics enables learners to make sense of data and understand the humanity behind the numbers (Gutierrez, 2010), to “read the world and remake it into a more just place” (Gutstein & Peterson, 2006, p. 6), and to achieve academic and professional skills that will enable them to critically participate in a multicultural society (Duncan-Andrade & Morrell, 2008).

**Statement of the Problem**

According to Hanushek, Peterson, and Woessmann (2010) in *U. S. Math Performance in global perspective: How well does each state do at producing high-achieving students?*, math performance of urban districts of low-income students compares to the math performance of schools in less developed countries representing an enormous step back for the US since the country should be compared to advance industrialized countries in the production of professionals as well as in providing and promoting equitable access to higher education. Statistical analysis of the results from standardized measures of academic achievement confirm the fact that students from high-poverty and high-minority areas are being pushed out of the educational system. The National Center for Education Statistics (NCES) reported in 2010 that “of the students who entered high school in the 2003-04 school year, 74% graduated within four years,
including 62% of Hispanics, 61% of American Indians/Alaska Natives, and 60% of Blacks” (p. v). In mathematics education, the NCES (2010) reported that the percentage of eighth-graders scored at or above Proficient was 18% American Indian/Alaska Native, 17% Hispanic, and 12% Black. Mathematics education of low-income children in the United States has taken a disengaged approach that eventually affects the students’ interest and disposition of the students to learn the subject. For example, only 62.3% of Hispanic freshmen students graduated from high school in the 2006-2007 school year (Common Core of Data [CCD], 2007). “In 2008, 55.7% of high school completers enrolled in two-year or four-year colleges” (Aud, Fox, & KewalRamani, 2010, p. 118).

To counter these numbers, it is key to understand that mathematics contributes not only to developing strong academic skills but also to critical thinking skills that help students, especially marginalized ones, to challenge, question, and evaluate evidence to unveil the hidden rules of access to social mobility and to change social disparities. The goal of math education is to provide a curriculum that respects the students’ background posing a variety of content-based real-life problems created by identifying cultural aspects of the community and later introducing them in the classrooms using teaching strategies that bridge the cultural differences between schools and communities; the strategies eventually have a profound influence on learning (Trumbul, Rothstein-Fisch, & Greenfield, 2000). In other words, teachers are challenged to construct curricula that draw upon the cultural resources students bring with them to the school (Scheurich, 1999). “Students must be involved in practices in which they are not only consumers of
knowledge, but producers of mathematical practices” (González, Andrade, Civil, & Moll, 2001, p. 130). It is important, then, for teachers to take their students’ languages, histories, experiences, and voices to create situations allowing them to learn, master, and apply math skills using real-life problems with a social justice foundation that reflects the students’ realities.

In *Pedagogy of the Oppressed* (1993), Freire stressed that men and the world exist together for which he urged teachers to bridge community and school by offering an alternative approach to the traditional teaching model in which the teacher teaches and the student listens, by respecting active dialogue with students and by providing them the means “to see the world not as a static reality, but as a reality in process, in transformation” (p. 83). In other words, it is about developing necessary critical thinking skills to be part of the world and to perceive themselves committed to participating in it. This Master’s project takes into consideration these ideas to develop an alternative/supplementary curriculum that would help balance the teaching of the subject, navigating from traditional teaching to an approach that introduces strategies by which students learn real applications of the subject and teachers are prepared to offer the tools needed by students to become independent critical thinkers.

*The Critical Math Social Project* consists of three parts. First, a theoretical framework is presented to teachers through professional development with the goal to guide them toward learning how to understand, internalize, and apply changes in their approach to teaching mathematics by using problem posing methodologies. It will also
provide an inviting and safe environment where they can become aware of and freely talk about social injustices and disparities, and, consequently, about the importance of offering critical mathematic social situations to their students. Secondly, a curriculum, focused on end-of-units projects will be developed based on Core Standards for sixth grade as a guide for teachers to use in their classrooms immediately after completing training. Third, follow-up meetings will be organized with the goal to provide support and encouragement to teachers to continue using social justice situations in their math classes. Through social justice situations, they will challenge students to use math concepts to learn about social injustices, power struggles, and educational inequalities, all of which can contribute to change social disparities and to unveil the hidden rules of access to social mobility. That is, the main goal of The Critical Math Social Project is to not only trigger social awakening in both teachers and students and develop academic math skills, but to also foster a safe and comfortable environment where issues about race, discrimination, environment, and lack of opportunities will be discussed and critiqued.

**Background and Need for the Project**

The social injustices of past schooling practices can no longer be tolerated.... Mathematics has become a critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate: Equity has become an economic necessity. (National Council of Teachers of Mathematics [NCTM] as cited in Croom, 1997, p. 1)
Historical Application of Mathematics

During the late 1970s and early 1980s, major concerns about the quality of the mathematics educational system in the US became apparent. Because the country was showing signs of falling behind Japan, West Germany, Eastern Europe, and Russia (USSR) regarding the production of engineers and scientists meaning imminent political and economical danger (National Commission on Excellence in Education, 1983), and because of the report of The National Research Council (NRC) (1989, 1990), “poor mathematic achievement by African American, Hispanic, Native American, and female students” (as cited in Croom, 1997, p. 1), a report called A Nation at Risk was developed by the National Commission on Excellence in Education (1983). The goal of this report was to create awareness and propose recommendations to change the route education was taking at that time period. As discussed by Ladson-Billings and Brown (2008), this recommendations included more days and hours of schooling, more academic courses, more attention to basics, more discriminating standards for evaluating and compensating teachers, more standardized testing of students achievement, and more elaborate reporting of test results not only at the K-12 schools’ levels, but also at colleges and universities.

Since then, mathematic reform has been proposed for implementation to move math instruction away from the traditional approach viewing knowledge as “discrete, hierarchical, sequential, and fixed toward a classroom in which knowledge is viewed as an individual construction created by the learner as he or she interacts with people and
things in the environment” (Draper, 2002, p. 521). Draper’s ideas are also supported by Boaler’s (2000) research in mathematics teaching when she conducted and presented the results of three-year longitudinal case study of children learning mathematics in two different schools: using a traditional textbook approach in one and using a curriculum focused only on open-ended projects in the other. The results of her study demonstrated “that a test-based, standardized curriculum unconnected to the real worlds of a mixed student population did less well than one that was deeply connected to the real world” (as cited in Connelly, He, & Phillion, 2008, p. 28). That is, the results of Boaler’s work (2000) at the school with a more traditional approach to teaching showed not only students’ confusion and inability to think mathematically in certain situations but also that they were unable to use their school mathematical methods in real-world situations. Thus, change from a traditional math teaching approach to a problem-posing education is well supported by theoretical as well as empirical research (Boaler & Staples, 2008; Cohen & Hill, 2000; DeLange, 1995). The speed of implementation of the appropriate changes has been slow despite the fact that it has been proven that when teaching gets farther away from the banking system, students are enabled to acquire high levels of math understanding.

**An Alternative to Traditional Mathematics Teaching**

Freire (1993) stated that the banking concept of education extends only to receiving, filing, and storing the deposits and called for a type of instruction centered on helping the students understand rather than just memorize math facts and operations
Therefore, the new approach to mathematics education should recognize the significance of the application of the subject giving students a sense that mathematics can be applied to real-life situations (NTCM, 1989). Unfortunately, the No Child Left Behind act (NCLB) of 2001 has pushed teaching mathematics to a frantic focus on mastering lower order skills using drill and practice (Erickson et al., 2008). As well, many schools concentrate resources and energy on measuring “state curriculum standards that often contain far too many expectations and address concepts and skills that are less than important” (Phillips, Reardon, & Yates, 2010, p. 75).

For the above reasons, Gutstein and Peterson (2006) argued that in most of the country’s math classrooms, the subject does not have real-life application and that “it is often taught in ways divorced from the real world” (p. 1). Consequently, to contrast the traditional approach, they proposed to rethink mathematics as “connecting math to students’ lives, linking math and issues of equality, using math to uncover stereotypes, and using math to understand history” (p. 11). In addition to this, Frankenstein (1983) argued, “knowledge of basic mathematics and statistics is an important part of gaining real popular, democratic control over the economic, political, and social structures of our society” (p. 1). Frankenstein’s concept makes the need to reject a traditional approach to teaching math that is usually “uninteresting, obsolete, and useless” (D’Ambrosio, 2007, p. 32) more urgent.

In some parts of the country, the sense of urgency for change has been routed to programs such as The Algebra Project created by Moses (2012) in the 1980s ensuring
high quality math education to low-income students so they can achieve college preparatory classes; the Interactive Math Program (IMP), a four-year problem-based math curriculum for high school; Connected Mathematics Project (CMP), a middle-grade curriculum that teaches through student-centered exploration; and Complex Instruction (CI), a form of cooperative learning that goes beyond promoting group work in public schools and makes students accountable for their education while learning how to build tolerance, acceptance, and patience toward different ideas and ways of learning. All the aforementioned programs promote change toward teaching mathematics in a manner ensuring students’ understanding of the real applications of mathematics, thus moving the subject toward a problem-posing approach that includes practices that “not only enhance individual understanding but also provide students with the opportunity to engage in practices that are represented and required in everyday life” (Boaler, 2000, p. 5).

**Benefits to Problem-posing Mathematics**

Cobb and Moses (2001) argued:

The new generation…can’t afford to be completely illiterate in math as they need these skills to function in the society, have economic viability, be in a position to…participate and have some saying in the decision making that affect their lives. (p. 14)

Cobb and Moses also stated in *Radical Equations: Civil Rights from Mississippi to the Algebra Project* (2012), that The Algebra Project “is not about simply transferring a body of knowledge to children. It is about using that knowledge as a tool to a much
larger end” (p. 15). In other words, all students can become academically competent in the subject when the methodologies chosen by educators promote a plan of action based in real-life situations. Moreover, these methodologies must be respectful of the students’ funds of knowledge, as their cultural and cognitive knowledge is validated in the classroom; therefore, students can be more easily engaged in their learning process. On the contrary, as argued by researchers such as Moll et al. (1993), “educational institutions…rather than focusing on the knowledge these students bring to school and using it as a foundation for learning, schools have emphasized what these students lack in terms of the forms of language and knowledge” (p. 4). There are many examples of teachers who have found positive results in their students’ math achievement by changing their traditional approach to teaching mathematics using real-life situations and the students funds of knowledge. “I used the family’s knowledge about owning and managing a store to create a math unit on money. For three weeks, we explored the social issues of money, along with mathematical concepts about money” (Moll et al., 1993, p. 13).

A progressive approach to teaching mathematics centered on making the subject more real to the students should have a strong foundation of Freire’s problem-posing education (1993) because he argued, “every thematic investigation which deepens historical awareness is thus really educational” (p. 109). The ideal problem-posing approach is based on teacher and students performing and carrying out, through dialogue, their roles with mutual collaboration. The main goal of this approach is to achieve a state
of personal growth that will allow students to “have the skills to function academically and professionally in a complex world and also the sensitivities to function as critical thinkers in a multicultural world” (Andrade & Morrell, 2008, p. 65).

Undoubtedly mathematics is a powerful academic tool that when primarily taught using strategies closely related to the students’ realities through the use of real-life applications, problem posing, it can provide “equal opportunity to acquire the quantitative literacy essential to compete for employment and leadership positions in today’s society” (Croom, 1997, p. 7). Therefore, mathematics taught with a social justice emphasis is a tool to provide and facilitate an environment in which students empower themselves to become socially aware to make sense of sociopolitical contexts and social disparities and to understand how the world functions. Hence, society benefits from critically educated citizens and citizens benefit from a critical approach to education. That is, societies need informed voices that demand change to stop historical cycles of social, financial, and environmental oppression and, therefore, discontinue disparities to overcome the stage of restraint that affects those citizens deprived of accessing equitable education. D’Ambrosio (2007) argued, “we do not want our students to become citizens who obey and accept rules and codes that violate human dignity” (p. 174). Above all, students should be able to understand that mathematics is more than solving a list of disconnected problems; it is about experiencing the subject as close to reality as possible. Thus, when confronted with issues in real life, students can apply their knowledge and consequently make decisions that can make a difference in their lives.
Teaching and Learning Mathematics for Social Justice

For many mathematics education researchers, an emphasis on the social has already begun, causing us to rethink such common terms as “learning” (Gutierrez, 2010).

Incomplete…

Problem-based mathematics. The importance of introducing problem-posing methodologies in the classroom has become clear in a reformed educational system that moves away from traditional education. Teaching and learning mathematics for social justice uses a problem-based approach that has a deeper impact on the students’ lives because it has meaning and provides a real-life application to the subject. When mathematics is taught using engaging strategies, students are motivated to become literate in the subject; therefore, they “are capable of interpreting the vast amount of quantitative data they encounter on a daily basis, and of making balanced judgments on the basis of those interpretations” (Schoenfeld, 1992, p. 4). Freida, one of Gutstein’s students, wrote in her journal:

I thought math was just a subject they implanted on us because they felt like it, but now I realize you could use math to defend your rights and realize the injustices around you…I mean now I think math is truly necessary and I have to admit, kinda cool. (Gutstein, 2006, p. 125)

Interdisciplinary. Another important aspect to this new approach is to teach mathematics across the curriculum by creating a close relationship with teaching and learning the social sciences. Issues presented in history curriculum can become math
lessons. Gutstein and Peterson (2006) stressed the importance of trying to highlight numbers that relate to social movements for equity and justice and also develop awareness of how the history of mathematics can point out the contributions of various non-European cultures and civilizations to mathematical thought, all of which offer a multicultural view of the subject. The idea is “to create positive feelings among students… promoting unity and tolerance in a society composed of different people” (Sleeter & Grant, 1988, p. 79). Some examples to be outlined in mathematics classrooms are the introduction of the concept of zero by Mesoamerican cultures, the contributions to arithmetic and geometry by Egyptian culture, and the Chinese culture’s contribution with the decimal positional notation system that presents early developments of the numeral system.

**Reading and writing.** The two components to learning of reading and writing should also be included when teaching the subject because mathematics can help students develop literacy and, most importantly, develop academic mathematics language, key to understanding the world, its social injustices, and how power has been disproportionally distributed.

**Limitation of the Project**

The implementation of social justice issues to achieve equity in mathematics is a complex process and a work in progress. First, well-trained teachers with a strong social commitment and sensitivity toward social disparities are necessary to reach a level of classroom discourse that encourages students to awaken and start the process of changing
their realities. Furthermore, to achieve continuity and consistency throughout a school, teachers need to continue working in collaboration with their peers and authorities as it takes effort, commitment, and perseverance to teach for social justice.

Second, as Ma (1999) stated, there is a deficiency of profound understanding of fundamental mathematics. Teachers’ mathematics professional backgrounds do not go beyond computational procedures. Ball, Hill, and Bass (2005) stated that knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably. This issue presents a real challenge for universities and school districts committed to teaching mathematics in context because most elementary school teachers have learned the subject through a banking education system encouraging a non-conceptual approach to learning mathematics with a total disconnection to real-world situations.

Lastly, NCLB’s strong emphasis on results in education has left no room for schools to move away from high-stakes testing and scripted packaged programs that take math education away from equitable methodologies promoting discussions, explanations, and the acceptance of diverse approaches to problem solving. In the year 2000, the National Teacher Council of Mathematics (NTCM) stated all students should have access to a curriculum that will allow them to achieve social mobility, all students should have access to high-quality curriculum and technology, and all students should have access to highly qualified teachers with adequate resources and subject-matter knowledge. Unfortunately, mathematics education is getting farther away from this goal as social
disparities promote oppression in education and leave marginalized students with fewer opportunities to learn, with less qualified teachers, and with fewer resources.

**Theoretical Framework**

**Critical Pedagogy**

The term critical pedagogy refers to a form of education centered on creating and promoting practices that encourage academic development and social awakening “within a context of social critique and struggle for social change” (Duncan-Andrade & Morrell, 2008, p. 21). In other words, Critical Pedagogy reflects on the importance of how to deliver content rather than what content to deliver, naturally without underestimating the importance of academics. It is “a pedagogy firmly committed to freedom and social change…to motivate students to develop sophisticated academic literacies” (Duncan-Andrade & Morrell, 2008, p. 51). Critical Pedagogy uses methodologies that promote production of knowledge and “learn how to learn” through social-oriented activities maintaining in mind the social issues or stratification of the school’s community. Could this critical approach be implemented in mathematics education? As Frankenstein (1983) argued, “knowledge of basic mathematics and statistics is an important part of gaining real popular, democratic control over the economic, political, and social structures of our society” (p. 1). Consequently, the need for an equitable approach to teaching mathematics in American schools today could guarantee the development of an emancipatory knowledge that helps make sense of
how social relationships are distorted and manipulated by relations of power and privilege. It also aims at creating the conditions under which irrationality, domination, and oppression can be overcome and transformed through deliberative, collective action. (McLaren as cited in Darder, Baltodano, & Torres, 2003, p. 64)

That is, the ultimate goal of critical mathematics is to develop and increase quantitative sophisticated reasoning to guide students to find empowerment to make “decisions in their personal life, on the job, and in matters of public interest” (Schoenfeld, 2002, p. 4), all of which closely tie with the main purpose of The Critical Math Social Project.

**Purpose of the Project**

What are the reasons behind children being more successful in mathematics in non-school settings, in other words, in the real world? When students are exposed to math in a non-traditional setting, they tend to understand the connection between the subject and their personal life, which is a key component to “doing” mathematics successfully. Battista (1994) argued that outside school, many children seem to use their intuition and conceptual understandings to decide what to do or what strategy to use. Thus, why can educators not have this same success in the classrooms? The fact that teachers are killing their students’ love for the subject tends to happen because math has historically been taught in an oppressive manner, with no real-life applications, disconnected from the students’ interests, and above all, using a one-size-fits-all curriculum.
The purpose of this project is to offer an alternative and supplementary approach to all the banking methodologies mentioned above with the creation of a set of standard-based mathematics lessons for elementary school students. It takes into consideration that teachers in charge of teaching the lessons need to move from a conceptual knowledge to a procedural knowledge. In addition to all the above mentioned, the lessons promote a strong teacher-student relationship focused on dialogue, equity, and social justice issues. The lessons provide academic mathematic knowledge so students:

- Become aware of social disparities
- Have the tools to achieve change/social mobility
- Embrace and accept cultural diversity
- Learn mathematics
- Appreciate mathematics.

And teachers:

- Become aware of their students’ realities
- Learn about conceptual knowledge
- Learn how complex instruction methodologies increase success of social justice education.

**Definitions of Terms**

*Banking education*

A concept of education in which the scope of action allowed to the students extends only as far as receiving, filing, and storing the deposits (Freire, 1993)
Critical Pedagogy

A process intended to be cyclical where students are encouraged to become social agents, developing their capacity to confront real-life problems that face them and their community (Duncan-Andrade & Morrell, 2008).

Culture

Particular ways in which a social group lives out and makes sense of its given circumstances and conditions of life (McLaren, 2009).

Generative themes

There are themes that generate discussion to get people more motivated to learn because the experience gives them insight into the power networks to which they are subjected (Freire, 1993).

Problem-posing education

The type of education that allows people to develop their power to critically perceive the way they exist in the world. It affirms men and women as beings in the process of becoming (Freire, 1993).

Social justice

Social justice pedagogy is that students themselves are ultimately part of the solution to injustice, both as youth and as they grow into adulthood. To play this role, they need to understand more deeply the conditions of their lives and the sociopolitical dynamics of their world (Gutstein, 2003).
Chapter 2

REVIEW OF RELATED LITERATURE

To surmount the situation of oppression, people must first critically recognize its causes, so that through transforming action they can create a new situation, one that makes possible the pursuit of a fuller humanity. (Freire, 1993, p. 47)

Purpose of the Review of Related Literature

This chapter represents overviews of the literature that helped ground or expand on the need for the Critical Math Social Project and present a critical analysis of mathematics education through a theoretical approach to the advantages and disadvantages of both traditional and critical mathematics. The first area looks at a critical analysis of mathematics in present day including teacher preparation and the role of the subject in a result-oriented era. The second area focuses on the importance of critical pedagogy leading to the need of a critical approach to mathematics and how the latter can improve the route teaching and learning mathematics has taken in the country. And finally, the third area investigated in this literature review is the idea of learning and teaching mathematics for social justice and the difficulties of it being introduced in the present educational system, including tension among teachers, administrators, parents, and students.
Critical Analysis of Mathematics in Present Day

Purpose of Mathematics Education

Teacher preparation. The National Council of Teachers of Mathematics (NCTM) issued recommendations in 2004 for the math community to explore and investigate the real influence of the “characteristics of school curricula that empower students from underrepresented groups to learn, [as well as the] cultural factors that influence mathematics teaching and learning, including analyses of the function of teachers’ worldview in the process of teaching and learning” (pp. 9-10). In other words, according to the NCTM, not only school curricula needed to be revised to enable access to equitable education by all students regardless of their skin color or socio-economic level, but also, the recommendations suggested the teachers’ subject matter competence be deeply analyzed, as their math qualifications can allow them to comfortably move from abstract to concrete mathematics and thus, guide their students to do the same. In this latter area, the NCTM (2004) in-depth recommendations included looking at the “teachers’ preparation, teacher induction, and [to add] mentoring programs, including alternative teacher certification programs, in regard to its effect on the mathematics learning of students from underrepresented groups” (pp. 9-10).

Such a support system is crucial especially for teachers whose beliefs about the nature of mathematics and how to learn and teach the subject is based on their previous experiences as students, polluted by the influence of prior teachers or math family history. On the other hand, teaching programs tend to perpetuate the concept that
mathematics needs to be learned and taught in a traditional manner, making the possibility of breaking the cycle of non-reform teaching more difficult. According to Shulman (1986), good teaching requires expertise in at least three areas: subject matter content knowledge; pedagogical content knowledge, knowing how to present the subject through questioning and flexibility so students can acquire more than formulas and logarithms; and also curricular knowledge, which relates to searching for alternatives for presenting the content. Teachers’ mathematics knowledge plays an important role in the success of teaching mathematics using a realistic approach requiring a deeper understanding of concepts and as argued by Kajander (2010), “teachers are generally unable to explain the methods they used, or provide a relevant model or example that might be needed for effective classroom teaching” (p. 248). In other words, teachers carry their own old approach to learning math and their fear of the subject into their classrooms, thus making their teaching superficial and in the form of facts and leaving behind the richness of mathematics as a tool to comprehend how the world works.

Ingersoll (2003) conducted a study to understand and explain the importance of the close relationship between teachers’ skills and students’ academic achievement. She concluded that the issue of under qualified or out-of-field teachers defined as teachers who have been assigned to teach a subject matter not necessarily matching their training or education has an impact and is reflected, for example, in the students’ math proficiency. For example, Wisconsin and Minnesota both have high levels of licensing standards for teachers, which correlates with their students’ high scores in national
assessments in mathematics, ranking numbers 14th and 2nd in the nation, respectively (NAEP, 2011). By contrast, “the comparable proportion of teachers with full state certification and a major in their field in Louisiana was only 64%” (Darling-Hammond, 1999, p. 16), thus reflecting on students’ math achievement ranking number 48th in the nation (NAEP, 2011). Darling-Hammond concluded in her study that “the less socially advantaged the students, the less likely teachers are to hold full certification and a degree in their field and the more likely they are to have entered teaching without certification” (p. 29). To support this study, it was recently confirmed by a NCES report that:

in 2007–08, about 25% of secondary mathematics teachers who taught in schools with at least half Black enrollment had neither a certification nor a college major in mathematics, compared to 8% of secondary mathematics teachers who taught in schools with at least half White enrollment. (p. 49)

Another example is the case of California in which minority students’ populations tend to receive instruction from under-qualified teachers. The Center for the Future of Teaching and Learning reported in 2005 that

when under-prepared, novice, and (in secondary schools) out-of-field teachers are counted together, it becomes clear that students in high-need schools are likely to have at least one teacher—or even a series of them—who is not fully prepared to help them succeed. (p. 98)

Figure 1 shows the results of the distribution of unprepared math teachers teaching minority students.
Source: The Center for the Future of Teaching and Learning (2005, p. 97)

Figure 1. Distribution of Underprepared teachers with a math assignment, by school-level percentage of minority students, 2004-05.

Also, according to Everybody Counts: A Report to the Nation on the Future of Mathematics Education (1989), “teachers of mathematics must have appropriate mathematical and pedagogical training” (p. 3) because “teaching for mathematical understanding is hard” (Schoenfeld, 2002, p. 20) and “the vast majority of today’s American mathematics teachers learned the traditional mathematics curriculum in the
traditional way” (Schoenfeld, 2002, p. 20). Moreover, out-of-field teachers or under-
qualified teachers teaching math have a skewed view of the subject that prevents them
from asking higher order questions and providing an environment in which students can
explore and investigate mathematical concepts to guarantee long-term learning of those
concepts. That is, out-of-field or under-qualified teachers “have neither models nor
experience teaching in the ways that would best facilitate their students’ development of
mathematical understanding” (Schoenfeld, 2002, p. 20) essential to develop the skills to
make sense of numbers. On top of the issue of teacher preparation there is the effect of
high-stakes testing as a training effect on student preparation. As argued by Amrein and
Berliner (2002), “the evidence presented here [study] suggests that in these instances
students are learning the content of the state-administered test and perhaps little else” (p.
58).

Not only the distribution of under-qualified teachers teaching mathematics has an
impact on the education of minority students. As it was reported by Esch et al. (2005),
“the historical patterns show that when there are too few qualified and experienced
teachers to go around, high-need students are shortchanged” (p. 98). Also, the excessive
use of high-stakes testing in schools today has turned teaching and learning mathematics
into a result-oriented type of education that has clearly had an impact on students’
acquiring a deep understanding of the subject as well as an impact on minority, low-
income, and special needs students’ education who have disproportionately failed test
requirements for promotion (Darling-Hammond, 2003).
Math Education in a Result-oriented Era

Education today places a strong emphasis on accountability for student performances via standardized tests. For example, regarding standardization McCoy (2011) concluded, “student’s success and worth comes from how well you do on those standardized tests and exams. ‘Intelligence’ boiled down to memorizing and being able to regurgitate facts unrelated to your situation, most often devoid of any critical thinking” (p. 5).

Under the federal No Child Left Behind Act of 2001 (NCLB), standardized test scores are the indicator used to hold schools and school districts accountable for student achievement. Each state is responsible for constructing an accountability system, attaching consequences—or stakes—for student performance. The theory of action implied by this accountability program is that the pressure of high-stakes testing will increase student achievement. (Nichols, Glass, & Berliner, 2005, p. i)

Many scholars argued students are “being tested with increased frequency, and the tests are being given greater weight than even before” (Duncan-Andrade & Morrell, 2008, p. 157). Boaler (1998) demonstrated there was little academic improvement in a test-based, standardized curriculum unconnected to the real worlds of a mixed-student population. And Ladson-Billings and Brown (2008) agreed by stating the NCLB act has left at-risk students with “weak standards, limited opportunities and soft bigotry of low expectations” (p. 163). Rhem (as cited in Smith, 2002), regarding students’ approaches to learning, also argued, “excessive amounts of material to be covered, lack of
opportunity to pursue subjects in depth, lack of choice over subjects and/or method of study, and a threatening assessment system target a surface learning” (p. 8) all of which are especially damaging for minority students.

The results of a study conducted over a three-year period by the Center for the Study of Evaluation (Briars & Resnick, 2000) in the Pittsburg public schools with a population of roughly 56% African American and 44% White/Other were conclusive at demonstrating significant disparities in minority students’ math performance in skills, concepts, and problem solving abilities. Pittsburg school district had about 100 schools at the time of the research, “some Pittsburgh teachers embraced the proposed reforms and some did not. This raises the question of whether there were performance differences between students who experienced the curriculum as implemented…and those that did not” (Schoenfeld, 2002, p. 14). Strong implementers used student-centered instruction and weak implementers used traditional mathematics instruction. Test results were compared in strong and weak implementation environments in different schools and the results confirmed that a change in the approach to teaching and learning the subject had an impact on the students’ achievement. According to Schoenfeld (2002), the results of his study “Making Mathematics Work For All Children: Issues of Standards, Testing, and Equity” showed, for example, that “in the strong implementation schools, the percentage of African American students meeting each of the concepts and problem solving standards is 30% or more, a more than seven-fold increase over their counterparts at the
weak implementation schools” (p. 15) compare to 4% of African American students meeting the assessed concepts in weak implementation schools.

![Bar chart showing percentage of 4th grade students in weak and strong implementation schools who achieved the skills, problem solving, and concepts standard in 1998.](image)

Source: Shoenfeld (2002, p. 16)

*Figure 2. Percentage of 4th grade students in demographically matched “weak implementation” and “strong implementation” schools who achieved the skills, problem solving, and concepts standard in 1998.*

This study confirmed the hypothesis about a strong relationship between a reform-oriented and traditional curriculum as the first approach to teaching mathematics.
shows better student results when measuring skills, problem solving, and math concepts. To confirm these findings Schoenfeld (2002) concluded:

The bottom line is that standards-based reform appears to work when it is implemented as part of a coherent systemic effort in which curriculum, assessment, and professional development are aligned. Not only do many more students do well, but also the racial “performance gap” diminishes substantially. (p. 17)

The problem is that since the introduction of No child Left Behind in 2001, education has moved away from reform-oriented curriculum toward a more test-oriented one, making schools districts make decisions to do things, such as implement scripted curriculum across grades. These types of programs are mainly being implemented in low-socioeconomic communities at schools considered “low-performing,” according to test reports provided by educational publishers. Low-socioeconomic schools are under the pressure to increase their tests scores and thus, avoid governmental sanctions, such as closures or decreased funding. Authorities see scripted programs as a technical fix to narrow the student achievement gap not only because they promote memorization and repetition, but because they make teachers’ teaching abilities and content uniform. Schoenfeld’s (2002) findings support the idea that a result-oriented education is not enough to enable students to succeed because:

Decisions affecting individual students' life chances or educational opportunities should not be made on the basis of test scores alone, and that alternative
assessments should be provided where test results may not provide accurate reflections of students’ abilities; that assessments should cover the broad spectrum of content and thought processes represented in the curriculum, not simply those that are easily measured; and that tests must provide appropriate accommodations for students with special needs or limited English proficiency. (p. 24)

In that same line of thought, the NCTM (2000) expressed concerns about the excessive use of testing on students, as it has been confirmed by Kohn (2000) that when teachers teach to the test, “both the content and the format of instruction are affected; the test essentially becomes the curriculum” (p. 19). What it is even more worrisome is under these circumstances, students fail to acquire critical thinking skills. As cited in Schoenfeld (2002), the NTCM recommended:

Assessment should be a means of fostering growth toward high expectations and should support high levels of student learning. When assessments are used in thoughtful and meaningful ways, students’ scores provide important information that, when combined with information from other sources, can lead to decisions that promote student learning and equality of opportunity. (p. 24)

Taking these recommendations into account and showing concern about at-risk students, scholars such as Stiff (2001) warned that when there is no improvement in “minority students' mathematics knowledge, skills, and problem-solving abilities [that creates] economic, social, and political disadvantages for these students as they advance into the future” (para. 1), making their chance to opt for higher paid jobs, for example, slimmer.
In other words, the National Center for Research (as cited in Shoenfield, 2004) argued the United States is at “risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond reach” (p. 263). That is, as disparities, opportunities, and educational inequity are becoming the status quo, the achievement gap between the rich and the poor, girls and boys, Whites and Blacks is becoming more pronounced and “unless corrected, innumeracy and illiteracy will drive America apart” (p. 14).

As explained by De Freitas (2008) one of the necessary changes in reformed education involves the application of real-life situations that connect to the students’ lives so “students are able to envision how they are implicated in the experiences of others, and how they might go about redressing the situation for the benefit of all” (p. 79). It is about acquiring “deep learning” (Rhem, 1995) where students are encouraged to interact with peers by working in groups in a well-structured knowledge base with connections of new concepts to prior experience and knowledge, with a strong motivational context, and with a choice of control and a sense of ownership. This is also supported by Briars and Resnick’s (as cited by Schoenfeld, 2002) study arguing that a change in the teaching approach might take mathematics education in the direction of providing equal access because “in 1997 roughly 10% of the ‘traditional’ students met or exceeded the standards for concepts or problem solving, while in 2000 roughly 25% of Pittsburgh’s (now ‘reform’) students met or exceeded those standards” (p. 13).
Also, to take some of these important facts about this new approach to teaching mathematics into practice, in 2010, the National Governors Association Center for Best Practices, Council of Chief State School Officers started creating a new set of educational standards called the Common Core State Standards for Mathematics. As stated by the Mathematics Assessment Resource Service (MARS), Common Core Standards require students to:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning. (AFT.org, 2012)

The new common core standards aim to take mathematics education away from a traditional approach, an issue that has been presented, studied, and acknowledged by the math community for some years now. However, there are some potential difficulties about the implementation of these new standards. For example, the NCTM (2004), in a joint statement with TODOS: Math for All, warned leaders and teachers that a “more holistic approach is required in addressing the CCSS efforts to close the achievement gap, as a focus on curriculum and assessment alone is not sufficient. Gaps in achievement are
an indicator of disparities between groups of students” (para. 1). They continued their statement by mentioning another important factor in the CCSS equation:

Biases are not explicitly addressed within professional learning experiences associated with implementation of the CCSS, perceptions of marginalized groups of students in U.S. schools as unable to learn rigorous mathematics may be reinforced. We believe it is time for renewed emphasis on equity as part of ongoing mathematics professional learning now and in the future. (para. 1)

That is, teacher preparation programs will need to be addressed to accommodate the new approach to mathematics, students will need new textbooks and materials that will guide them to acquire these new and “different” standards, and more time and resources will have to be assigned in classrooms especially those that house diverse and low socioeconomic students.

Beyond doubt, math education needs a change, a reform, in its approach to teaching and learning that over many years has been studied, researched, and recommended to mathematics organizations, teachers, parents, authorities, and communities. However, it seems as though what is also needed besides a radical change in the subject’s instruction is one in the perception that school and society are separate entities. According to Goodlad and Oaks (1998), “school exists as part of a larger ecosystem that often hampers the schools’ efforts to become a renewing culture in which the very best educational and social values permeate daily life” (p. 22). Thus, to achieve
the goal of a more liberatory math education, what is essential to include in schools today is a critical approach to teaching and learning mathematics.

**Critical Approach to Mathematics**

**Intersection of a Critical Pedagogy and Math Education**

Burbules and Berk (1999) argued that the critical person is one empowered to seek justice, to seek emancipation. Not only is the critical person adept at recognizing injustice but for Critical Pedagogy, that person is also moved to change it. Forty years have passed since Paulo Freire (1993) shook the educational world with his strong and radical thinking about human relations, pedagogy, and critical thinking in his book *Pedagogy of the Oppressed* where he argued, "[Education] becomes the practice of freedom, the means by which men and women deal critically and creatively with reality and discover how to participate in the transformation of their world" (p. 81). He highlighted the idea of a revolutionary approach to education through problem posing instead of banking education. Educators desperately needed to achieve social changes through cooperation, participation, and knowledge.

Other scholars, such as Frankenstein (1983), were inspired by Freire’s ideas. Frankenstein stated the importance of a different approach to education, arguing, “liberatory social change requires an understanding of the technical knowledge that is too often used to obscure economic and social realities” (p. 1). She made mathematics education her main focus for social research and argued that academic content is as important as students gaining social freedom necessary to exit a stage of submission,
oppression, and ignorance. In other words, as research proved, the context of education should be focused on developing critical thinking skills, as learners and teachers need to become aware of political and social decisions that affect their positionality in society.

According to Freire (1993), the role of education should be a revolutionary one where students and teachers can “see the world not as a static reality, but as a reality in process, in transformation” (p. 83). It should be one in which they can have active influence by dialoguing with teachers to break a cycle of banking education, which opposes “garner[ing] knowledge and skills that students can put to work in real life roles…that require appreciation, understanding, and judgment” (Perkins, 1993, p. 3). This role of education is supported by the work of Frankenstein (1983) in which she argued mathematics could enable students to work together in “uniting reflection about [for example] statistics with action for social change” (p. 14). On the contrary, as discussed by the mathematics community, rote knowledge does not provide education for the future, it does not give “students a sense that mathematics is a subject that is applied to real life” (Center for the Study of Mathematics Curriculum, 2004, p. 4), perpetuating the idea that “mathematics is utilitarian…not a tool to read the world” (Gutstein, 2006, p. 31). Otherwise stated, with this approach to mathematics, students develop the skills to understand how the world works and ensures the development of a type of sociopolitical consciousness that builds the connections, allowing students to be aware of situations that are unjust, deceiving, and oppressive. Consequently, it can create new situations via
what Freire (1993) calls “transforming action” so they can pursue a “fuller humanity” (p. 47).

Educating for critical consciousness, conscientizacao, requires students to be capable of questioning, being critical, and capable of becoming problem solvers so they can have access to understanding the dynamics of their world and consequently change. That is, an education that “includes the roles and responsibilities of students, the pressures on teachers, the complexities of moving students from massified to critical consciousness, and the tenuousness of the link between an emerging critical consciousness and radical social change” (Frankenstein, 1983, p. 17). Paulo Freire introduced the idea of consciousness or conscientizacao in the educational community about 40 years ago. Its importance will be analyzed as a separate component of education.

Conscientizacao – Developing Critical Consciousness

As reported in Freire’s (1993) book *Pedagogy of the Oppressed*, “the term conscientizacao refers to learning to perceive gender, social, political, and economic contradictions, and to take action against the oppressive elements of reality” (Translator’s note, p. 35). For example, the disparities related to women’s access to more complex mathematics can be analyzed, studied, and discussed to create awareness of gender disparities in mathematics education because, as concluded by Frankenstein (1983), “not only can math skills and concepts be learned in the classroom from applications which challenge the hegemonic ideologies, but interested students can also work with the many groups uniting reflection about statistics with action for social change” (p. 14). She also
called for math content that challenged the students’ realities through math context problems or *generative themes* as Freire called them.

Freire and Frankenstein argued about the importance of content that promotes citizenship, expressing concerns about “content presented in a nonliberatory way…cannot challenge students’ reality and cannot inspire commitment to radical change” (p. 6). Change is a key component to accessing citizenship through an education based on critical pedagogy. Consequently, moving toward critical thinking requires students to develop this kind of consciousness as the process of accessing liberating education because as Freire (1993) concluded, consciousness consists of acts of cognition not transferals of information. How can educators ensure reaching levels of consciousness in their students? It is crucial to establish a pedagogy that respects the student’s realities, nurtures critical thinking, and delivers high quality content; that is, a problem-posing pedagogy. If problem-posing pedagogy is disregarded by using a pedagogy that centers on depositing information, then fewer students will “develop the critical consciousness which would result from their intervention in the world as transformers of that world” (Freire, 1993, p. 73). Programs presented by Duncan-Andrade and Morrell (2008) in “The Art of Critical Pedagogy” are examples of teachers “developing students’ sense of critical self-consciousness through shared reading such as *Savage Inequalities, Hagakure*, and *The Art of War*” giving students the opportunity to “understand that they, together with other youth in their community, could be agents of change” (p. 81). Another example is the work of Gutstein and Peterson (2006) of a math
book with problems centered on social themes they believe have a strong relationship
with issues of equality, stereotypes, and history. “Driving while black or brown,” “The
war in Iraq: How much does it cost?”, “Environmental hazards: Is environmental racism
real?” are a few examples.

The work of many scholars such as Frankenstein (1983) on critical mathematics
literacy education; Gutierrez (2007) on equity, success, and the future of mathematics
education; Croom (2008) on developing an approach to mathematics for all students; and
De Freitas (2008) on recognizing the ethical dimension of problem solving. All indicated
the role of mathematics education should be the “practice of freedom” where students
exist as a reality in the world (Freire, 1993) and where they acquire more than traits and
abilities needed to contribute to the growth of the economy with more validation,
participation in the gains, and retribution with social or health securities. For example,
Schoenfeld (2001) confirmed the idea of the practice of freedom or literate citizenship, as
it
calls for making a plethora of informed decisions—about interest rates, about
situations that are inherently probabilistic, about the nonsense spewed by
politicians. The best way to learn to make sense of applied situations, and to learn
to assess claims made by others, is to have lots of practice building and assessing
mathematical models. (p. 53)
Such practices would allow students to wonder, critique, and acquire a critical viewpoint
of societal information and misinformation with a strong academic mathematical support.
All the above reasons outline the importance and need of a critical approach to mathematics.

**Critical Mathematics Education**

Giroux (2010) argued, “critical pedagogy offers the best, perhaps the only, chance for young people to develop and assert a sense of their rights and responsibilities to participate in governing, and not simply to be governed” (p. 1). He called for the need to develop critical thinking and levels of consciousness in students as it takes into consideration the student’s interests and motivations to engage them in self and social empowerment (Giroux as cited in Darder et al., 2003). Such an approach to teaching, recognized by Freire (1993) as problem-posing education, can also be applied to teaching and learning mathematics. Critical mathematics education provides not only the tools to “prepare the students for further learning and more effective functioning in their lives” (Perkins, 1993, p. 2) but also provides “assisting students to draw on their own voices and histories as a basis for engaging and interrogating…experiences that provide them with a sense of identity, worth, and presence” (Giroux as cited in Darder et al., 2003, p. 453). That is, students question what is being presented to them, teachers provide instances in which students can develop awareness for further “life awakening,” and they work through dialogue to find answers to situations that represent their real-life struggles.

Thus, as Freire (1993) argued the use of generative themes is key to the success of critical mathematics as they help to develop a culturally relevant approach to the subject where students can study “the positive aspects of conflict, such as its role in promoting
creative change and in bringing attention to injustice, students will develop the critical insight that society is not static” (Apple as cited in Frankenstein, 1983, p. 8). However, this is not an easy task, as Croom (2008) suggested changing the approach to teaching mathematics requires “extensive modifications of practices, schools structures, policies, curriculum, pedagogy, and assessment” (p. 7), a more complex challenge to achieve in schools today, as they tend to perpetuate social disparities. According to De Freitas (2008), these “mechanisms of cultural reproduction that continue to sustain systemic inequity in/through education cannot simply be named and then easily abolished” (p. 81).

Thus, scholars agree that problem posing has a deeper and positive impact on students’ education than a traditional or banking system because the core of problem-posing education falls into the concept of praxis that leads to students and teachers reflecting on situations that aim to pursue change through the use of dialogue and generative themes. Duncan-Andrade and Morrell (2008) developed the image in Figure 3 to highlight the steps educators need to follow to successfully apply a problem-posing approach.
Unfortunately, there are still too many teachers using a banking system or a traditional approach to teaching mathematics in classrooms around the country, but especially in those containing more vulnerable students. For example, Boaler’s research in the year 2004 supported that approximately 21% of the time in some algebra classes was spent with teachers talking to the students, usually demonstrating methods; approximately 48% of the time students were practicing methods in their books and working individually. All the above proved approximately 97% of teacher’s questions were focused on procedures rather than facilitating students’ discourse and exploration. To counter the above findings, she also observed that when teachers used strategies that,
as argued by Perkins (1993), asked students to “explain, muster evidence, find examples, generalize, apply concepts, analogize, and represent in a new way” (p. 5), students enjoyed math more than those at more traditional schools and the achievement gap was smaller than that at more traditional schools. The goal of critical math education is to provide access to quality mathematics and guide students to make sense of the subject to eventually become aware of its real-life immediate applications. As a student in Boaler’s (2004) study explained:

A math person is a person who knows like, how to do the work and then explain it. Like explaining everything to everyone so they could get it. Or they could explain it the hard way, the easy way or just, like average – so we could all get it. That’s like a math person I think. (Jorge, Y1, p. 9)

Other studies such as Confrey and Kazak’s (2006) have also focused on mathematics teacher practices arguing a vast majority of mathematics teachers predominately serving children from low socioeconomic backgrounds still teach skill and drill. These studies concluded that more vulnerable children do less well at mathematics than children from privileged homes because of the teachers’ beliefs that they need their basic math skills rather than a more holistic approach to the subject through posing real-life problems that help develop positive mathematical attitudes and allow students to gain control over a subject as a tool to understand the world. The work of Boaler (2002) also supported these findings of the belief that marginalized students need a more structured approach to learning mathematics. “Teachers simply believed that students did not
receive the support at home that they needed to cope with work that was linguistically and conceptually demanding so they provided them with more structure to help them” (p. 255). To a certain extent, structure or direction has benefits in mathematics education. For example, it is necessary to guide students to start a discourse about proposed math problems. However, when used in excess, students’ math learning becomes short-term, dependent, less flexible, disconnected from the real world, and all about memorizing a set of rules and algorithms. Draper (2002) cautioned math teachers about not creating a “math classroom that invites inquiry, initiates students’ questioning” (p. 522), which later reflects in students’ lack of opportunities for higher education and in their disinterest in social issues directly affecting their lives. Sadly enough, Gates (2006) also strongly discussed the fact that the literature sufficiently convincingly shows this is an international trend.

The preoccupation of an immediate change of approach to critical mathematics is highlighted in the research of Skovsmose and Borba (2004) in which they argued that such an education is the route that needs to be followed because its primary concern is:

The social and political aspects of the learning of mathematics; provides access to mathematical ideas for everybody independent of color of skin, gender and class; sees mathematics in practice, being an advanced technological application or an everyday use; as well as citizenship values that have to be developed in the classrooms as they represent a democratic forum, where ideas are presented and negotiated. (p. 207)
There is consensus among the math community that a reformed math classroom should provide students the “opportunity to explore and discuss as they construct their mathematical understanding” (Draper, 2002, p. 522) using methodologies that enable them to gain knowledge of mathematics and statistics. All of these are important goals to achieve to ensure a “real popular, democratic control over the economic, political, and social structures of our society” (Frankenstein, 1983). That is, while it is about changing methodologies, it is also about revising or rewriting content in a manner meaningful for the students. It can help them “become subjects capable of using critical knowledge to transform their world” (Frankenstein, 1983, p. 6). This can be achieved, for example, by taking the students’ cultural backgrounds to build context problems that have some meaning for them. As argued by Murrell (1997), teaching mathematics also includes helping students develop positive social and cultural identities by validating their language and culture and helping them uncover and understand their histories. Put differently, when students’ language and cultural backgrounds are respected and taken into consideration when posing a problem of their interest, then their chances at success in mathematics increases. Ladson-Billings (1995) concluded that culturally relevant pedagogy should promote academic success, cultural competence, and develop a critical consciousness through which students can challenge the status quo of the current social order.

Mathematics teachers should promote a sense of citizenship in their classrooms by developing a curriculum that is culturally respectful and meaningful. For example, De
Freitas (2008) proposed that teachers can easily modify the mainstream textbook on a daily basis by revising questions that “address the student in terms other than profit and consumerism” (p. 84). Hence, changing math education to a problem-posing approach in which method and content go together to create a “connected curriculum that equips and empowers learners” – or better said, “nurtures an environment where the students empower themselves” (Perkins, 1993, p. 14) – “for the complex and challenging future they face” (Perkins, 1993, p. 14).

Above all, an important element to achieve success in critical mathematics education is dialogue. Much has been talked discussed regarding dialogue in the educational world, as it helps to redefine the student-teacher relationship. Gutierrez (2010) mentioned, “through dialogue, learners are given opportunities to express themselves and act on their knowledge. In this way, students are offered a greater number of choices in how they can interact as citizens” (p. 5). Additionally, Frankenstein (2010) stated, “through constant searching and dialogue, we can continually refine our understanding in the sense that we can act more effectively” (p. 3). And Freire (1993) wrote, “dialogue, as the encounter among men to ‘name’ the world, is a fundamental precondition for their true humanization” (p. 137) because through dialogue education becomes powerful, it turns into a revolutionary tool with which students can “transform their realities” (p. 126).

Teaching and learning mathematics with social justice are essential to terminate the exclusion of a segment of our society that has been oppressed through education over
many generations. Hence, it is critical to provide educational access and equity because as Volmink argued (as cited in Skovsmose & Borba, 2004), mathematics cannot serve as a “gate keeper to participation in the decision making process of society” (p. 2). In other words, the goal of democratic mathematics can only be achieved when students are able to acquire a level of competency that allows them to make sense of numbers, ask questions that require them to think critically, and approach math problems without any anxiety or fear, i.e., to become math literate or quantitatively literate.

Critical Mathematics and Literacy Development

The idea that everything in mathematics is either right or wrong causes many students great difficulty because they prefer to think in shades of gray, not just black or white. This helps explain why mathematics serves as a “critical filter” that blocks students with weak mathematical skills from rewarding careers. Just how important is it that all students master formal mathematics? Might context-rich quantitative literacy be a more reasonable alternative? (Ewell, 2001, p. 38)

Quantitative literacy. Quantitative literacy (QL) involves not only mathematics known as an abstract science that contains numbers, symbolism, and proofs, but also the ability to make sense of the numbers because “mathematics making-sense makes a difference in the real world” (Schoenfeld, 2001, p. 51). It is about being able to draw conclusions from limited information or numbers with confidence and to acquire an independence necessary for making decisions; in other words, the goal is to focus on inductive and analogical reasoning rather than deductive reasoning in order to really
understand real-world phenomena (Manaster, 2001). Mathematicians have agreed and supported the importance of developing QL in mathematics classes; for example, Cohen (as cited in Steen, 2001) argued QL:

is required to understand important political debates on issues such as Social Security funding differential effects of various tax-reduction plans, and health insurance options…only few Americans have the quantitative savvy to work through these policy debates and evaluate all their implications. (p. 27)

Otherwise stated, the education system has consistently failed to provide math academic literacy and has not supported critical pedagogy to ensure an education that goes beyond information and facts. Other scholars have also warned that ignoring the issue of elevating the subject to analyzing, explaining, asking questions, wondering, and conversing could keep mathematics performance and competencies relegated to basic skills and memorization of facts. For example, Schoenfeld (2001) discussed that QL includes “confidence with mathematics; a cultural appreciation of mathematics; the ability to interpret data, to think logically, to make decisions thoughtfully, to make use of mathematics in context” (p. 51) all of which can only be achieved by seriously rethinking and reshaping math education. When mathematics is delivered in context, studies such as the Pittsburgh report, Duncan-Andrade and Morrell’s (2008) work in urban schools, and The Algebra Project (2012), to name some examples, have shown that test scores increase and students’ attitudes toward the subject become more positive, anxiety-free, and realistic. With the introduction of CCSS for mathematics, students and teachers will
be challenged to learn and teach a new form of math that is unknown and frightening for many of them.

The challenge is huge, but the opportunity to introduce culturally relevant activities and curricula will be wide open, as well as the chance to develop students’ critical thinking through contextualized math social justice problems posed to them. Opportunities will be created for discourse and academic development that would challenge their oppressive views of their world. In other words, the ultimate goal is to develop QL in all students. Some concrete examples of what QL includes Ellis (2001) visualizing that QL:

may include reconciling a bank statement, analyzing data to support or oppose a local government proposal, estimating how to split a lunch bill, debugging a program by working from assumptions toward a logical conclusion, deciding which medical treatment to pursue based on statistical evidence, building a logical court case, or understanding the risks in investing for retirement. (p. 63)

All the examples would allow students to question their positionality in the world and to eventually become active participants in the modern world.

**Teaching and Learning Mathematics for Social Justice**

The need to give mathematics a social justice approach has become a regular topic within the education community. This approach is new to researchers and teachers and it has endured criticism that portrays it as “increasingly superfluous, complicating, and even threatening by some policy makers and pressure groups who increasingly see
any curriculum not tied to basic literacy or numeracy as disposable and inappropriate” (Michelli & Keiser as cited in Hytten & Bettez, 2011, p. 8). In spite of this criticism, the truth is that equity and opportunity for all students can be accomplished in mathematics education by making the necessary changes to math content.

According to De Freitas (2008), one such change involves the application of real-life situations that connect to the students’ lives so “students are able to envision how they are implicated in the experiences of others, and how they might go about redressing the situation for the benefit of all” (p. 79). Real-life situations, or social justice themes, can then help reestablish a sense of belonging, cooperation, respect, and responsibility in all classrooms. Teachers’ participation to support this new approach to mathematics is key, as they need to be prepared to “pose questions to students to help them address and understand [sociopolitical] issues” (Gutstein, 2003, p. 3). They need to develop a genuine understanding of how important it is for students to become conscious of their role in society to ultimately be able to believe in “themselves as people who can make a difference in the world, as ones who are makers of history” (p. 4). According to Freire (1993), every human being, no matter how ignorant or submerged in the culture of silence he or she may be, is capable of looking critically at the world, transforming realities, and creating culture and history in a dialogical encounter with others. Everybody can learn, exist, and create. Hence, it is morally right to provide equal access to mathematics education. Access leads to math achievement – prestige and higher salaries – with a genuine appreciation of the student’s roots and a truthful respect of their
identities so they can see themselves as part of the world and know they count. Consequently, they will be empowered and awakened to fulfill their role in society (Gutierrez, 2007). But are teachers ready to shift their beliefs leaving behind tension to co-investigate social issues with their students in math classes?

**Tension**

A growing number of researchers argue that critical pedagogy can support the ongoing struggle for equity in mathematics education (Frankenstein, 1995; Gutstein, 2003; Skovsmose & Valero, 2002; Tate, 1995). However, teachers’ inclusion of social justice issues in their lessons is one of the key components to ensure the success of this approach. Bartell’s (2011) research showed that tension arose among teachers as:

A discussion emerged around the issue of opening up one’s classroom to the sometimes controversial and discomforting topic of racism. Pat asked early in one session whether the group wanted to go into “real personal [issues] like racism or you know some ‘ism’ of some sort? (p. 15)

The new role of well prepared and committed teachers requires true generosity and honesty to understand different forms of social oppression as well as to objectively provide instances for dialogue in their classrooms. Grant and Gillette (2006) suggested teachers needed to be culturally responsive in the classroom to know themselves and be open to change, to hold a well developed philosophy of education, to maintain an educational psychology that is multicultural, and to connect teacher education to the outside world. Tensions arise not only when teachers have to bring social topics into the
classrooms and engage students in these discussions but also when teachers realize teaching for social justice requires students’ understanding of math concepts rather than students memorizing rules to succeed on standardized tests. However, tests are a reality in the education systems today, and, therefore, when developing problems, the relationship between the social topics and the mathematics to be used has to have a strong correlation with the standards. This idea is reinforced by the work of Gregson (2011) as she observed that when social justice was introduced in class through context problems, teachers lost more control of class time devoted to preparing for the test and also struggled to teach for conceptual understanding. She also concluded there was some difficulty trying to “mathematize social justice problems and teach students to explore them” (p. 29).

It had been well documented that new and in-service teachers do not feel prepared and perhaps they feel scared and apprehensive about introducing social justice topics in their classes. This is also accentuated by the fact that their teaching preparation did not include training in a realistic approach to the subject. Therefore, they started their teaching experience with no theoretical background and no practice. Moreover, there is the issue of teachers’ beliefs. A study conducted in the Netherlands by Wubbels, Korthagen, and Broekman (1997), who worked with student teachers to bring awareness to their “deeply engrained notions, feelings, values, attitudes and behavioral tendencies related to the teacher’s image” (p. 20), reported mix results. The research concluded it was difficult training teachers to change their perceptions because this endeavor “requires
the search for techniques and strategies in teacher education which induce important changes in the student teachers’ conceptions of mathematics and mathematics teaching, conceptions which are deeply engrained” (p. 24).

This is the new challenge in education today: working with new and in-service teachers who have a strong relationship with the banking system and guiding them to explore and perhaps eventually change beliefs so they explore and implement new approaches in teaching and learning mathematics. It is about promoting critical thinking in educators as well as developing a strong academic foundation. One idea of how to approach this challenge was proposed by Wubbels et al. (1997) that through reflection, student teachers were exposed to analyzing their own teaching. The goal of the study was to produce teachers who would ultimately be capable of independently tracing a process consisting of the following phases:

- Action: confrontation with a concrete and real situation which requires action;
- Looking back on the situation and the action;
- Awareness of essential aspects;
- Creation of alternative solutions or methods of action;
- Trial.

Human beings are capable of reflection and dialogue. Healthy communication cultivated among teachers, administrators, and community members have shown positive results in many schools that should be an example for other schools to follow.
Dialogue

**Dialogue and the teacher student relationship.** Dialogue is when two or more people willingly commit to resolve issues affecting both sides through true, open, and healthy conversation (Freire, 1993). Humans are conscious beings; thus they have the ability to express themselves and make powerful statements. Humans and animals experience the world differently. Humans can discern, while “animals lack the ability to exercise limit-acts, which require a decisive attitude towards the world” (Freire, 1993). That is to say, dialogue is the foundation for any human relationship; and the word is the foundation of profound, meaningful, and powerful dialogue. When teachers and students are ready to openly discuss meaningful issues using the right selection of words in a respectful manner, and with a humbled attitude, then the journey to reach liberation as partners in the educational system has been initiated. “Liberation is a praxis: the action and reflection of men and women upon their world in order to transform it” (Freire, 1993, p. 79). The role of dialogue is crucial in teaching and learning for social justice. Without the appropriate use and selection of words, students will continue living in a stage of dominance as they have not been given an education that promotes the acquisition of tools enabling them to confront the cycle of oppression. Consequently, equity of achievement in all social areas could become a reality.

**Equity.** Much has been said about equitable education, but recorded progress in this area of education has been marginal. Croom (2008) noted that disproportionate numbers of African American, Hispanic, Native American, and female students still lack
levels of acceptable math achievement. Gutierrez (2007) argued, “equity means fairness, not sameness” (p. 2), therefore, content and students’ backgrounds—socioeconomic status, gender, and race—are interrelated to achieve an equitable education. That is, according to Gutierrez (2007), what we need is “more research on effective/successful teaching and learning environments for black, Latina/o, First Nations, English language learners, and working class students” (p. 15). To support this idea, Croom (2008) concluded “an equitable learning environment affirms the richness of cultural diversity and creates an opportunity to engage all students in an interactive learning process” (p. 4).

In other words, equity is about providing opportunities for meaningful math learning, but above all, it is about “students’ relationships with each other developed through mathematical work as well as students’ relationships with the subject of mathematics” (Boaler, 2004, p. 14). The classroom then turns into a micro democratic system where students and teachers work together. The teacher models behaviors that respect other ideas and cultural backgrounds; consequently, students learn and take responsibilities that can be carried to their homes and communities. That is, in a classroom where equity is effectively promoted, teachers actively “involve students in learning and classroom decision making and set clear academic and social goals” (Grant, 2006, p. 297). Students are now critical investigators learning their positionality not only in the educational process at their schools, but also learning it in society. Achieving these
types of goals is not an easy task; however, there are classroom strategies teachers can use to promote successful teaching and equitable learning.

For example, Boaler’s study in Complex Instruction (CI), a type of group occurrence that goes beyond cooperative learning in diverse classrooms, included practices such as multidimensional classrooms, roles, assigning competence, and teaching students to be responsible for others’ learning process as well as their own. Otherwise stated, students worked in groups with specific roles such as facilitator, team captain, recorder/reporter, or resource manager, which provided students with a sense of belonging and responsibility for attaining their groups’ academic goals as well as diminishing math anxiety for low-achieving students. Another important component of CI observed was “teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group’s attention” (Boaler & Staples, 2008, p. 25).

It was an approach that provided all students with the opportunity to be valued by the work they do well. Justifying in mathematics, or showing one’s work, not only fulfills the goal of encouraging understanding and math discourse, but also provides students the opportunity to feel equally valued, as individuals, in a classroom environment. Complex Instruction was “designed to counter social and academic status differences in classrooms, starting from the premise that status differences do not emerge because of particular students but because of group interactions” (Boaler & Staples, 2008, p. 21) and the results of the study showed that the approach contributed to ensure
students’ higher academic achievement. The big picture is to ensure educational mobility to all students as it helps diminish economic disparities and social friction and increase personal happiness. This can be achieved by educators who have high expectations, are effective and inspirational, have strong leadership, and use a curriculum focusing on enabling students to become active participants in society.

**Summary**

The review of literature investigated the need for a different approach to teaching and learning mathematics. For many years, the subject has been portrayed as needing delivered rules, arithmetic, and memorization of logarithms without understanding the importance of a more critical, realistic, and social approach to the subject. Three areas were investigated from the analysis of mathematics in the present day, teachers’ preparation, the role of the subject in a current results-oriented era, the importance of a critical approach to the subject, and above all, the need and difficulties of introducing teaching and learning mathematics with a social focus. These three areas highlighted the importance of taking into account a change of approach as well as a change of teacher perceptions to accomplish change in the students’ understanding of the subject as well as its real-life applications that would eventually impact their participation in society.

Not only are effective leaders needed in classrooms across the country, but also in all top positions in education related jobs. Teachers, principals, and other authorities have the challenge to empower and engage students in the education process. Change is needed to help especially those more vulnerable students so they can access better
vocational job opportunities or higher education through a four-year college and to
decrease dropout rates. Seven thousand students drop out of school every day, one every
26 seconds (Wooldridge, 2010), 42 million Americans cannot read, write, or perform
simple math (National Right to Read Foundation, n.d.), 90% of the jobs offered in the
country require a high schools diploma. Change is needed to modify a curriculum into
one that “teaches higher-level quantitative reasoning skills in a more applied and
accessible context in which the goal is both knowledge and understanding” (Carnevale &
Desrochers, 2003, p. 27). It is also needed for an outcome of better prepared teachers
who are diversity-sensitive, inspirational, committed, content knowledge experts,
cooperative, curious, and open minded, some of the qualities the country needs in its
classroom leaders today.

The indisputable and empty teacher-student relationship based on banking
education shows an urgent need for this system to be rethought, redirected, and
eventually replaced by a more democratic approach as methodologies supported by the
former keep students in a stage of alienation, conformity, disconnection, oppression,
ignorance, and poverty. Freire (1993) argued, “students-no longer docile listeners-are
now critical investigators in dialogue with the teacher” (p. 81) representing a new
definition of the role of the students in the educational process. However, this dialogue
cannot be achieved if students persist in maintaining acceptance of the dominant rules,
are resistant to achieving confidence, and do not hope their participation in their process
of humanization is essential for an education as a practice of freedom. Students’
awakening through a problem-posing education with an emphasis on social justice issues is a personal journey; thus, the key is for teachers to provide them with the tools to ultimately understand social disparities, to perceive their realities, and to take action. As stated by Freire (1993), the individual can gradually perceive personal and social reality as well as the contradictions in it and mathematics can serve as a means to acquire a deeper understanding of society and the world enabling the integration and participation of all.
Chapter 3
DEVELOPING THE SOCIAL MATH LITERACY PROJECT

Introduction

The purpose of this chapter is to present an overview of the professional development that outlines and details the conceptualization and implementation of The Social Math Literacy Project. The focus of The Social Math Literacy Project is on solving mathematics problems that are open-ended based on the principles of Realistic Mathematic Education (RME). Using the key components of this educational approach, such as questioning, reasoning, and discussions, as well as the presentation of a theoretical frame, attendees will have the opportunity to experience solving RME problems. Later in the professional development, participants will go through the transformation of these open-ended problems to context problems with an emphasis on social justice. In other words, this professional development is a progression; a journey that will take the participants from understanding the significance of mathematics presented in the form of an everyday real-life problem to a different and more profound social real-life context that will eventually allow students to question their positionality in the world.

The goal of modifying RME problems is to guide participants through questioning and discussions about the impact of social issues in their students’ present and future lives. Otherwise stated, these “new problems” will have a social critical thinking approach giving mathematics the opportunity to become a “human activity”
(Freudenthal, 1973), which includes the use of applied mathematics, offering the opportunity for open discourse or dialogue that was the main focus of Freire’s problem-posing education.

This progression or professional development will take place over a period of three sessions allowing participants to experience solving Realistic Mathematics Education problems. After a clear understanding of Freudenthal’s theoretical framework of this approach to teaching mathematics, the RME problems will be changed into social justice math problems. The change of the norm math problems will be done during the second session of the professional development by introducing the importance of social justice in today’s classrooms through the philosophy of Paulo Freire. Finally, in the third session, participants will be able to develop their own bank of social justice problems, and, therefore, become independent learners who may apply the ideas learned during the professional development with confidence and enthusiasm. The Social Math Literacy Project will provide ample opportunities for participants to reflect, brainstorm, and dialogue about mathematics and issues concerning them the most.

**RME Theoretical Framework and General Guidance**

**Realistic Mathematics Education**

Realistic mathematics education (RME) was chosen to become the primary source of context problems as a problem-based curriculum that provides a connection between the students’ world and the mathematics content needing to be learned. That is, context problems are the instrument to be used in classrooms to guide and help students achieve
math academic knowledge and an education of freedom that ultimately will help them become competent citizens. According to Austin (2001), “the shift towards more open-ended, practical tasks provided an opportunity for collaborative exploration” (p. 24) and RME provides ample opportunities for students to explore and learn math from peers.

The founder of RME Professor Hans Freudenthal envisioned and worked for a more problem-centered or contextual approach to mathematics stressing that the subject is a human activity. That is, he believed mathematics must be connected to reality, stay close to children, and be relevant to society in order to be of human value. Thus, RME presents a sense of urgency to change the mainstream approach to teaching and learning mathematics into a life-long learning experience in which the use of mathematics is a tool enabling students to make sense of the world in which they live and interact on a daily basis.

**Why a Realistic Approach to Mathematics?**

The changing conditions of the structure of the world have awakened scholars to rethink the traditional, mainstream, or banking system of teaching and learning mathematics to an approach that has its foundation in real-life problems, as societies have evolved from agricultural to industrial and from industrial to a knowledge economy (Carnevale & Desrochers, 2003). For example, the Georgetown University Center on Education and the Workforce (Carnevale, Smith, & Strohl, 2010) reported the following projections on the requirements of work growth through the year 2018 and educational levels:
• Only 19 states will be at or above the 63% of jobs (nationally) requiring a postsecondary education beyond high school in 2018.

• Jobs in the District of Columbia will have the highest concentration needing postsecondary education in 2018.

• The highest proportions of Bachelor’s degree jobs and graduate degree jobs will be concentrated in the northeastern states.

• Jobs for workers with some college or with Associate’s degrees will be dispersed throughout the country.

• Jobs for high school graduates or dropouts will be concentrated in the southern states.

• Three states, Arkansas, Louisiana, and West Virginia, are more than 10 percentage points below the national average, which means the educational composition of jobs in these states will be mostly high school education levels or lower in 2018.

The results of the report are not new to the educational world or the international community. Carnevale and Desrochers (2003), in their article *The Democratization of Mathematics*, mentioned, “the shift toward a high-skilled service economy [that] required more and better integration of quantitative and verbal reasoning abilities. Problem solving in high- skilled service jobs is embedded in complex social interactions that mix both quantitative and verbal reasoning” (p. 28). Also, Ellis (2001) argued:
Mathematicians need help to develop curricula that provide students opportunities to be involved both in abstract thought and practical problem solving. I embrace real-world problems…because they can engage students in the abstraction, generalization, and logical thought that are the lifeblood of academic mathematics. (p. 64)

In the same line of thought, Steen (1999) acknowledged the need to provide the academic skills needed to succeed in this “new” world through well selected real-life problems, as people need to cope with:

Data, graphs, and statistics both enrich and confuse our lives. Numbers and quantities overwhelm current events, from medical reports to political trends, from financial advice to social policy. News is filled with charts and graphs, while quantitatively based decisions control education, health, and government. (Steen, 1999, para. 1)

Above all, the goal of a realistic approach to teaching and learning mathematics is to accomplish “common sense and to think logically…the nation now requires that its citizens and workers have the ability to reason in a commonsense way in situations involving numbers, graphs, and symbols” (Ellis, 2001, p. 61). That is, data have been analyzed and made public so now it is the educational system’s responsibility to enable students to acquire skills to compete in the job market.

“Students indicated they found the approach more interesting and relevant than when taught under traditional methods” (as cited in Schoenfeld, 1992, p. 24). According
to Schoenfeld (1992) “the mathematician best known for his conceptualization of mathematics as problem solving, and for his work in making problem solving the focus of mathematics instruction, is Pólya” (p. 16). Pólya saw “mathematics as an activity” (as cited in Schoenfeld, 1992, p. 16) and as an “active engagement of discovery” (p. 17) through the use of open-ended problems. Pólya’s book *How to Solve It* (as cited in Schoenfeld, 1992) presents four major challenges open-ended problems present to students:

a. Understanding the problem. What is the unknown? What are the data?

b. Draw a figure.

c. Devise a plan. Have you seen it before? Go back to definitions.

d. Carry out the plan.

e. Look back. Can you check the result? Does it make sense?

There is agreement that contextual meaningful math problems are needed to ensure equitable implementation of the new standards. The NCTM (Strutchens & Quander, 2011) recommended “teachers must incorporate worthwhile tasks that engage all students in thinking about and making sense of mathematics, not just practicing concepts and procedures they have already been taught” (p. 3). The Council also suggested reasoning and sense making should be organized in four categories: “analyzing a problem, implementing a strategy, seeking and using connections, and reflecting on a solution” (p. 4). CCSS for mathematics calls for a new approach to teaching and learning mathematics focusing on the importance of not only reasoning, but also in perseverance,
planning, attending to the meaning of quantities, and justifying conclusions, all of which are necessary for students to be “successful in our fast-paced, economically competitive society will increasingly require innovation and creativity” (Strutchens & Quander, 2011, p. 2).

**Professional Development – Journey to Critical Mathematics**

The goal is for the participants to understand and experience how Realistic Mathematic Education can be implemented in math classes and how open-ended problems benefit students’ understanding of mathematics. Open-ended problems allow students to “find the intended mathematics by him or herself…allowing the students to build their own mathematical knowledge” (Gravemeijer & Terwel, 2000, p. 786).

Participants will develop an understanding of how the use of the RME approach enables students to use models, reason mathematically, make sense of problems, and work collaboratively in groups. Gravemeijer and Terwel (2000) follow-up with some reasoning behind using RME being because “doing mathematics was more important to Freudenthal than mathematics as a ready-made product” (p. 780).

The work of Garet, Porter, and Desimone (2001) supports the idea of grouping teachers by schools during professional development. First, grouping teachers by schools enables them to work as a team to discuss and support the activities they need to implement in their classrooms; and second, these authors argued, “focusing on a group of teachers from the same school, professional development may help sustain changes in
practice over time, as some teachers leave the school's teaching force and other new
teachers join the faculty” (p. 922).

The first action is participants in *The Social Math Literacy Project* will be
grouped by their sites or by grade level as this will allow them to create connections
among them and be able to follow up with their future social justice lessons.

**Complex Instruction**

Complex instruction is an important component of the professional development,
as it models the type of teacher-student and student-student interaction-relationship that
needs to happen in math classrooms to create an environment of collaboration,
multidimensionality, where many different abilities are valued and respect. Complex
instruction is supported by Gravemeijer and Terwel (2000), as they argued that working
in groups will enable students to experience that when

Comparing and discussing their solution methods, for instance, some students
may realize that other solution methods have advantages over their current
method. This crucial role of dialogue as applied to interpretations, ideas and
methods once more shows that an emphasis on mathematizing does not only
imply solitary activity on the part of the individual student. (p. 783)

Participants will be assigned with a role in their groups: facilitator, team captain,
recorder/reporter, or resource manager. Participants will be given a math activity that
will allow them to learn about each other as well as learn about the role each one will
perform in their groups. Attendees experience complex instruction – group work with a
purpose – by working in the following activity: participants will have to learn about their roles and used them properly in the process of working in the activity. They will experience CI throughout the professional development. The presenter will model CI, as they will when they work in the grouping activity and with the RME problems. The purpose of the grouping activity is to have participants experience CI with a more relaxed and friendly activity. The “heavy problems” come later; it is a warm-up activity.

The participants will see a graph with one circle representing each member of the group. They will complete the graph by finding two different variables that will relate the four of them. When the activity is completed groups will have time to brainstorm the following question:

*Why do you believe that complex instruction is necessary to promote equity and access in all mathematics classes?*

The logic behind having them discuss complex instruction directly is to make it metacognitive and make a direct connection between what they are doing in the professional development and why they will be doing it with their students. It is critical to do this as the in-service goes on rather than after. Deconstructing what they are learning and making those connections need to be done in process so meaning is not lost (Brown, 2004). After brainstorming the question in their groups, participants will share their thoughts with the rest of the participants. The presenter will guide their contributions so the following principles of complex instruction become clear: it promotes multidimensional classrooms, it uses roles, it assigns competence, and it
teaches students to be responsible for each other’s learning. Participants will receive a handout on Complex Instruction and Boaler’s study (see Appendix A). At this point in the professional development, participants are ready to be challenged with a math problem that will serve as a tool to introduce Realistic Mathematic Education and its theoretical framework.

**Challenge to Participants: RME Problem**

The goal of presenting participants with the RME problem is to have them experience the same feelings, challenges, questions, and problem solving strategies their students will go through with similar problems presented to them in their math classes. It is important for teachers to experience feelings such as wondering, perseverance, frustration, risk-taking, willingness thinking about unfamiliar problems, creativity, and sense making, to name a few.

I will model a strategy called the 3Rs, modeled by Harold Asturias at the CMC 2012 conference. The researcher has used it in her class and students seem to follow the one-ended problem. Three people have to read the problem, one at a time, before the group starts working on it. Allow time for the groups to discuss and solve the problem.

**RME problem**

*A group of friends rent for a weekend camping farm. A week before the trip 4 friends had to withdraw from the trip.*

*The remaining participants will therefore pay $5 per person more.*
Then at the beginning of the weekend, two more friends cannot make the trip and let the group know that they will not pay.

As a result, the remaining friends have to pay $3 extra each.

How many friends were there and eventually how much money did they pay?

Groups will share different approaches to solving the problem to transition to RME principles and key aspects of RME. Presenter will show a possible solution using models, a geometric approach and then, a solution using algebra.

A possible approach to finding the solution to the problem is horizontal mathematizing or sharing other possible, interesting solutions. This would be done after they share their own different approaches to solving the problem. It is interesting and useful for math teachers to see how other teachers think and how they approach solving the problem.

Another possible solution to the problem is algebraic or utilizing vertical mathematizing.

1. There are \( x \) number of friends. Each will have to pay a total of \( y \) dollars.

   Therefore, these friends together will pay \( xy \) dollars.

2. If there are 4 friends that drop out, then there will be \( 4y \) less money for the trip.

   Then, \((x - 4)\) will be the number of friends who will pay \( 4y \) extra dollars in total.

3. Since each remainder friend has to pay $5 extra, the remaining friends will pay \( 5(x - 4) \) dollars in total. Then, \( 4y = 5(x - 4) \)

4. Since 2 more friends cannot go, then the remaining friends will have 2 \( (5y) \) less money for the trip (or money that needs to be shared by the remaining friends).
Now, the remaining friends represented by \((x - 6)\) will have to add another $3 to the original amount.

5. At this point, the remaining friends will have to pay an extra amount of \(2(y + 5)\) dollars. Therefore, \(2(y + 5) = 3(x - 6)\)

6. Using a system of equations, \(4y = 5(x - 4)\) and \(2(y + 5) = 3(x - 6)\)

7. \(x = 36\) There are 36 friends going on the trip

8. Each friend paid $40, \(y = 40\)

9. They all paid \(40 \times 36 = 1440\)

When sharing the above solutions of the problem is complete, participants will share their personal experiences in their groups about how they worked on this math problem, the kinds of personal and group challenges they faced, and the mathematics behind the problem through brainstorming the questions below. With the questions asked, participants can be guided to explain and verbalize their experiences of solving the problem. Some participants will have had similar or different experiences to each other. Exploring experiences continues the problem-solving learning process and helps concretize the process. The teachers should have an easier time bringing RME to their students due to their own processing of it.

**Brainstorming questions to groups:**

- What was your first approach to solving the problem?
- What was your experience when solving the problem?
- Did you use models to solve the problem?
• Does this approach fit on: teaching pure mathematics and afterwards showing how to apply it?

• Could you teach content that appears in the problem and has been covered in class?

After brainstorming and reflecting in the above questions, the presenter will write participants’ responses and help organize their ideas to later conceptualize the RME problem principles below. When the problem principles have been discussed, brainstormed, and agreed upon, participants will participate in group discussions to generate the key principles of RME through questions including those regarding the role of students and teachers in RME.

_Brainstorming questions to groups:_

• What is the key component of RME?

RME is realistic

• How does RME allow students to solve problems using different approaches?

Through the use of models

• Inductive or deductive? Explain

Inductive

• What is the role of students in RME?

Students discover and experience mathematics

• What is the role of teachers in RME?

Teachers facilitate and model
RME Problem Principles

1. It is not about this one problem, it is about mathematics
2. Promotes class discussions
3. Present open assignments
4. Encourage complex problems
5. Teachers try to understand the students’ reasoning
6. There must be ample opportunities to try things and think about the result
7. Keep the big picture in view
8. Provide new opportunities: return to fundamental issues
9. Establish relationships with previous lessons

Activity principle. RME gives students the opportunity to actively participate in their mathematics learning processes. According to Freudenthal (as cited in Van den Heuvel-Panhuizen, 2000), “the students, instead of being receivers of ready-made mathematics, are treated as active participants in the educational process, in which they develop all sorts of mathematical tools and insights by themselves” (p. 5). In other words, mathematics should be presented to the students in a manner in which they can investigate and produce their own understanding of the content, as this process ensures understanding of the subject rather than short-term memorization of concepts.

Reality principle. In Dutch, the word zich realiseren means to imagine, and so the term “realistic” refers to situations that can be imagined (Heuvel-Panhuizen, 2003). RME initially presents knowledge within such a concrete context, allowing pupils to
develop informal strategies, but gradually through the process of guided
“mathematization” and allowing students to progress to more formal, abstract, standard
strategies (Note: these contexts are chosen to help students' mathematical development,
not simply because they are interesting!). As Bell and Shiu (as cited in The University of
South Dakota, n.d.) suggested, “Abstract relationships are expressed by symbol-systems,
and rules are developed for the manipulation...meaning can only be restored to the
manipulations by recognizing the underlying concepts” (para. 20). It is when no meaning
is offered that misconceptions arise. The RME problems, set in real-world contexts, are
presented so that along with giving meaning and making mathematics more accessible to
learners, they also illustrate the countless ways in which mathematics can be applied
(para. 18).

Mathematization principle.

Horizontal mathematization. This is suggested by Harrison (as cited in The
University of South Dakota, n.d.) to be the students' discovery of mathematical tools that
can help organize and solve a problem located in a real-life situation.

Vertical mathematization. This refers to the building up to create more
challenging mathematics, leading to a greater use of abstract strategies.

Ex: The chickens and rabbits problem. Show solving it with pictures, then looking
for patterns to finally move toward algebra.

Use of effective models principle. Using various models is key in RME as the
models “serve as an important device for bridging this gap between informal, context-
related mathematics and more formal mathematics” (Van den Heuvel-Panhuizen, 2000, p. 5) that will eventually take the students to a level of vertical mathematization. To fulfill the bridging function between the informal and formal levels, models have to shift from a model of a particular situation to a model for all kinds of other, but equivalent, situations. An important requirement for having models functioning in this way is they are rooted in concrete situations and they are flexible enough to be useful in higher levels of mathematical activities. Thus, the models will provide the students with a foothold during the process of vertical mathematization without obstructing the path back to the source. Examples of models are ratio tables and combination charts.

**Guidance principle.** As indicated by Gravemeijer et al. (as cited in University of South Dakota, n.d.), this principle implies beginning with the range of informal strategies provided by students and building on these to promote the materialization of more sophisticated ways of symbolizing and understanding. Due to students directing the course of lessons, RME requires a highly “‘constructivist’ approach to teaching in which children are no longer seen as receivers of knowledge but the makers of it” (Nickson cited in University of South Dakota, n.d., para. 18), and the role of the teacher is that of a facilitator. Allowing the students to begin with the basics, using informal strategies and constructing the mathematics for themselves, simulates the discovery of the mathematics and allows them to appreciate the complexity of the mathematics.

**Interaction principle.** Education should offer students opportunities to share their strategies and inventions with each other. By listening to what others find out and
discussing these findings, the students can get ideas for improving their own strategies. Moreover, the interaction can evoke reflection, which enables the students to reach a higher level of understanding. Figure 3 shows how knowledge and deep understanding are closely related when students are given the opportunity to interact and engage in math dialogue. It describes the importance of adding students’ interaction and dialogue in math classes to ensure deep understanding.
The goal of the afternoon session is to give participants the opportunity to identify the big ideas covered, the strategies and the models that the students will use to solve a problem. They will use the RME problem solved in the morning session. The three elements of big ideas, strategies, and models are from Fosnot’s approach to teaching mathematics and it can be found in Context for Learning Mathematics. Fosnot’s approach will enable teachers to find common ground on how to develop units at their own sites in a straightforward and simple manner. The presenter will provide handouts and time for group discussion. Participants will have time to read the handout with a summary of Fosnot’s ideas in their groups (see Appendix B). They will have time to
have a group discussion to later complete a table with information about the RME problem presented earlier and then, they will have time to share their thoughts with the rest of the participants. They will also receive a copy of the CCSS for mathematics because they will need them to find the big ideas and strategies in problems.

Table 1

*Strategies for Solving Traditional Problems*

<table>
<thead>
<tr>
<th>RME problem</th>
<th>Big Ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Create equations that describe numbers or relationships</td>
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<th>Strategies</th>
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<tbody>
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<tr>
<td></td>
<td>• Rearrange variables to highlight a quantity of interest</td>
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<tr>
<td></td>
<td>• Solve systems of linear equations</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>RME problem</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Table</td>
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</table>

**Learning Mathematics as a Tool to Become Quantitative Literate**

Mathematicians have agreed and supported the importance of developing quantitative literacy in mathematics classes; for example, Cohen (as cited in Steen, 2001) argued QL:

is required to understand important political debates on issues such as Social Security funding differential effects of various tax-reduction plans, and health
insurance options…only few Americans have the quantitative savvy to work through these policy debates and evaluate all their implications. (p. 27)

The social math literacy project gives participants the opportunity to achieve and experience the transition from RME to a social justice approach to mathematics. The idea of giving mathematics a social approach is to enable students to understand the “socio-political, cultural-historical conditions of one’s life, community, society and world…to contribute meaningfully to social change” (Gutstein, 2006, pp. 24-27). This will be accomplished through the gain of a theoretical foundation on Freire’s problem-posing education with the ultimate goal of guiding students to become critical thinkers and engaged citizens. Teachers need to develop a genuine understanding of how important it is for students to become conscious of their role in society to ultimately be able to believe in “themselves as people who can make a difference in the world, as ones who are makers of history” (p. 4). This session will start with a warm up activity to give participants the opportunity to reflect on the ideas behind RME presented in Session 1 and be able to move toward social justice mathematics. They will reflect and talk in their groups to later share their thoughts and comments with the rest of the groups. They will go through the same process of brainstorming they learned from Session 1 and will share their thoughts with each other as they did in Session 1. Moving through this exercise will bring them closer to social justice mathematics.
Finding the Relationship Between Maslow’s Hierarchy of Needs, Gardner’s Theory of Multiple Intelligences, and Freudenthal’s RME Principles

After the above activity is completed, the presenter will take the theory of Maslow: Hierarchy of Needs, Gardner’s Theory of Multiple Intelligences, and Freudenthal’s RME principles to start making the connection to Freire’s critical pedagogy. Groups will discuss and reflect on questions regarding the three theories, which will allow them to scaffold the main ideas behind these three theories and their connections to teaching mathematics. Modeling scaffolding shows teachers that students need structure but with some flexibility to explore open-ended mathematics problems. It is what Freudenthal called “guided reinvention,” about giving students a “contextual problem that allows for a variety of solution procedures…preferable solution procedures that in themselves reflect a possible learning route” (Gravemeijer & Terwel, 2000, p. 786). Participants will make a diagram relating the three theories. The three theories are important because: a) Learning occurs when needs are met, b) Not all students are mathematicians, and c) Mathematics is a human activity. Making the diagram uses

When this activity is completed, each group will share their thoughts, their findings, and their diagrams about teaching mathematics and the theories of these three scholars. The following theoretical framework explains the theory of Maslow: Hierarchy of Needs and Gardner’s Theory of Multiple Intelligences.
Maslow: Hierarchy of Needs

Teachers must provide a safe and inviting learning environment where students feel comfortable emotionally and psychologically to achieve academic competence. The goal is for students to take control of their learning processes, which can be achieved by the teacher fostering respect, tolerance, and high expectations. When teachers take into account the students’ realities, they are not only fulfilling students’ psychological needs such as belonging and esteem, but they are meeting self-fulfillment needs, such as allowing students to achieve full potential (Maslow as cited in Simons, Irwin, & Drinnin, 1987). Maslow emphasized the following 10 points on his hierarchy of needs:

i. We should teach people to be authentic, to be aware of their inner selves and to hear their inner-feeling voices.

ii. We should teach people to transcend their cultural conditioning and become world citizens.

iii. We should help people discover their vocation in life, their calling, fate or destiny. This is especially focused on finding the right career and the right mate.

iv. We should teach people that life is precious, that there is joy to be experienced in life, and if people are open to seeing the good and joyous in all kinds of situations, it makes life worth living.

v. We must accept the person as he or she is and help the person learn their inner nature. From real knowledge of aptitudes and limitations we can know what to build upon, what potentials are really there.
vi. We must see that the person's basic needs are satisfied. This includes safety, belongingness, and esteem needs.

vii. We should refresh consciousness, teaching the person to appreciate beauty and the other good things in nature and in living.

viii. We should teach people that controls are good, and complete abandon is bad. It takes control to improve the quality of life in all areas.

ix. We should teach people to transcend the trifling problems and grapple with the serious problems in life. These include the problems of injustice, of pain, suffering, and death.

x. We must teach people to be good choosers. They must be given practice in making good choices. (Simons, Irwin, & Drinnien, 1987, p. 2)

Teachers will receive a handout containing information on Maslow’s needs (see Appendix C).

Gardner’s Theory of Multiple Intelligences

It is also important to have a discussion with participants and have them reflect on Gardner’s Theory of Multiple Intelligences, as the theory reminds us that not all students “have the same interests and abilities, not all of us learn the same way” (Gardner, 1999, p. 10). In other words, teachers (and people, in general) tend to easily forget that not everyone has a logical-mathematical intelligence or is able to think logically or be able to solve numerical patterns. Teachers need to keep an open mind and be sensitive to not only the fact it is important to develop strategies to create an environment that is inviting
to all to learn math, but also it is important to develop other intelligences in their classrooms, such as interpersonal and intrapersonal, both necessary to function on personal levels as well as on social levels. By scaffolding with these two theories, Maslow: Hierarchy of Needs and Gardner’s Theory of Multiple Intelligences, introducing Freire’s problem-posing education, developing social justice mathematic problems will be easier.

Introducing Freire’s Ideas

Two main ideas will be introduced in this section of the session: problem posing education and generative themes. RME’s principles, activity, reality, and interaction are key to Paulo Freire’s problem-posing education. The above three principles will be linked to educational philosophy. Participants will brainstorm about what they already know about the concepts in their groups. “Students must be involved in practices in which they are not only consumers of knowledge, but producers of mathematical practices” (González et al., 2001, p. 130). Thus, scholars agree problem posing has a deeper and more positive impact on students’ education than a traditional or banking system because the core of problem-posing education falls into the concept of praxis that leads to students and teachers reflecting on situations aiming to pursue change through the use of dialogue and generative themes. Duncan-Andrade and Morrell (2008) developed the steps educators need to successfully apply a problem-posing approach. The steps are a) identify a problem, b) research the problem, c) develop a collective plan of action to address the problem, d) implement the collective plan of action, and e)
evaluate the action, assess its efficacy, and re-examine the state of the problem. This professional development also follows their steps as the teachers move through the activities and evaluate and assess each section of the process. The teachers will answer questions regarding problem-posing education and assess how they might use it in their classrooms. The presenter will guide discussion to answering the questions to have a group agreement on the problem-posing theoretical framework. The theory included in the Critical Pedagogy section is for the reader as well as for participants who will receive a handout with this information (see Appendix D).

**Critical Pedagogy**

Paulo Freire argued education should be “the practice of freedom” and he strongly believed the traditional approach to education, “the practice of domination,” would not help achieve this goal. He proposed an alternative method: Problem-posing. The core of problem-posing education falls into the concept of praxis, “the process by which teachers and students commit to education that leads to reflection on that action” (Duncan-Andrade & Morrell, 2008, p. 24).

Freire (1993) described the importance of generative themes to ensure the success of problem-posing education, as they are key elements to engage students so they can become actively involved in the education process. Generative themes are social, cultural, economic, or political issues educators promote in their classrooms, bringing them to life through mathematics and strictly of the students’ interests. It is about bringing the student’s knowledge and interests to the classroom through activities and
projects reflecting their realities because, as González et al. (2001) stated, this knowledge is often not in their textbooks but is acquired from the streets, family cultural traditions, youth culture, and the media.

**The Bridge: From Freudenthal’s Ideas to Freire’s Ideas: Moving Toward Social Justice Mathematics**

Presenter will give time to participants to refer back to the diagram of Maslow’s theory and Gardener’s theory they developed earlier to analyze where Freire’s ideas could be included. The participants will share their thoughts about their diagrams. After this interactive dialogue. The focus of the section is to show participants how the theories behind RME and problem posing can harmonically intersect in math classes, which stresses the point that social justice can be used in math classes. This activity is also about stressing the point that RME can provide the basic mathematical components to create problems to which the students can relate in a manner enabling them to become productive citizens. According to Gutstein (2003), teachers “need to develop a genuine understanding of how important it is for students to become conscious of their role in society to ultimately be able to believe in “themselves as people who can make a difference in the world, as ones who are makers of history” (p. 4).
Table 2

*Comparison of Freudenthal and Freire Theories of Education*

<table>
<thead>
<tr>
<th></th>
<th>H. Freudenthal</th>
<th>P. Freire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics is a human activity</td>
<td></td>
<td>Education should be the practice of freedom</td>
</tr>
<tr>
<td>Against deductive approach to teaching math</td>
<td></td>
<td>Against banking education</td>
</tr>
<tr>
<td>Use of real life problems</td>
<td></td>
<td>Use of generative themes</td>
</tr>
<tr>
<td>Mathematics as an activity</td>
<td></td>
<td>Problem posing education</td>
</tr>
<tr>
<td>Mathematical discussions</td>
<td></td>
<td>Dialogue</td>
</tr>
<tr>
<td>Group work- Collaboration</td>
<td></td>
<td>Group work- Collaboration</td>
</tr>
<tr>
<td>Teacher as a facilitator</td>
<td></td>
<td>Teacher as a facilitator</td>
</tr>
<tr>
<td>Students have an active role in their learning process</td>
<td></td>
<td>Students are not docile listeners</td>
</tr>
<tr>
<td>Applied and pure mathematics are including in real life problems</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At this point, participants will have an understanding of the importance of student participation, the inclusion of themes, and the incorporation of dialogue in their math classes. The presenter will shift focus onto the math problems containing a social justice theme that can be used in their classes. This is the beginning of modifying RME problems into math problems with a social justice emphasis.
The Theory Behind Social Justice Problems

The types of problems posed in *The Social Math Literacy Project* are open-ended social justice problems, as the combination of problems with a meaningful social approach and those promoting curiosity lead to students becoming productive citizens. This type of problems not only provide the tools to introduce discourse giving students the opportunity to explain with their own words, justify their thinking, create awareness, and find solutions to situations that closely affect the students’ lives. The problems also provide opportunities to discover mathematical relationships and make sense of them. As stated by Gardner, “in the end, education has to do with fashioning certain kinds of individuals—the kinds of persons I (and others) desire the young of the world to become. I crave human beings who understand the world, who gain sustenance from such understanding, and who want—ardently, perennially—to alter it for the better” (Gardner, 1999, p. 5).

The New Problems

The presenter will bring back the problem solved in the first session and with the participants, revise the mathematics behind the problem using Fosnot’s ideas. The purpose of this activity is to give participants time to work in their groups and transform problems into social justice problems.

*A group of friends rent a camping farm for a weekend. A week before the trip 4 friends had to withdraw from the trip.*

*The remaining participants will therefore pay $5 per person more.*
Then at the beginning of the weekend, two more friends could not make the trip and let the group know they would not pay.

As a result, the remaining friends had to pay $3 extra each.

How many friends were there and eventually how much money did they pay?

Modifying an RME problem to a problem with a social justice theme. Three different versions of social justice problems will be presented and teachers will reflect on five different questions that will guide them to modify their own problems in the future. Examples of social justice math problems that are a modification of the above RME problem:

A company’s tight budget calls to finish a project in a set number of hours. The company hires temporary workers to complete the assignment. Before they begin the job, the manager fires 4 workers and 5 more hours are added to each worker left. Just before they start the job, the manager fires 2 more workers and 3 more hours are added to the remaining workers. How many extra hours will each worker work? How many hours are assigned to finish the project?

Questions to groups:

- What is the theme?
- What is the purpose of this problem?
- Can you expand more to move students towards their conscientization?
- What kind of dialogue can you have in the classroom with this topic?
- Further questions from this problem?
The purpose of these questions is first to identify the topic the math problems bring for discussion, and then understand why this topic is important for the students’ critical approach to how the world functions. Further questions can relate to more specifics such as why hire temporary workers, which relates to retirement, health benefits, vacation. Also, it relates to employers’ excessive use of offering temporary employment and workers’ rights. More examples of questions like the above are in Chapter 4, the curriculum.

Example: Completing the table below using the modified RME problem presented in session 2:

*A company’s tight budget calls to finish a project in a set number of hours. The company hires temporary workers to complete the assignment. Before they begin the job, the manager fires 4 workers and 5 more hours are added to each worker left. Just before they start the job, the manager fires 2 more workers and 3 more hours are added to the remaining workers.*

*How many extra hours will each worker work?*

*How many hours are assigned to finish the project?*

Table 3 completes Table 1 presented in previous sessions, as it includes social justice topics to which teachers might be open for class discussions. It helps teachers organize their math social justice problems and prepare for any dialogue the problem could create in the classroom.
Table 3

**Example Template to Organize Social Justice Problems**

<table>
<thead>
<tr>
<th>Path to Literacy and Social Justice Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Big Ideas</strong></td>
</tr>
<tr>
<td>• Create equations that describe numbers or relationships</td>
</tr>
<tr>
<td><strong>Strategies</strong></td>
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</tr>
<tr>
<td><strong>Models</strong></td>
</tr>
<tr>
<td>• Table</td>
</tr>
<tr>
<td><strong>Social Justice Themes</strong></td>
</tr>
<tr>
<td>• Temporary workers</td>
</tr>
<tr>
<td>• Hiring benefits for temporary workers</td>
</tr>
<tr>
<td>• Benefits for employers when hiring temporary workers</td>
</tr>
<tr>
<td>• Health benefits for workers?</td>
</tr>
<tr>
<td>• Wages</td>
</tr>
<tr>
<td>• Retirement plans?</td>
</tr>
</tbody>
</table>

**Path to Literacy and Social Justice Mathematics**

The goal of this segment is for participants to experience modifying RME problems into social justice problems. They will have access to a list of RME problems as well as different social justice topics. Participants will be challenged to not only solve math problems, but to dialogue about social justice issues that can be used to modify the
presented problems. Session 2 ended with the presentation of one RME problem modified into three different social justices problems. By discussing five questions, participants were guided to find the social themes and the purpose of the problem to move students toward conscientization. At the beginning of this session, participants will review RME in their groups, along with social justice math, and Fosnot’s ideas as a warm up activity.

**Collaborative Lesson Planning**

This activity involves planning, teaching, observing, and critiquing the lessons. While working on a study lesson, teachers jointly draw up a detailed plan for the lesson, which one of the teachers uses to teach the lesson in a real classroom (as other group members observe the lesson). The group then comes together to discuss their observations of the lesson. Often, the group revises the lesson, and another teacher implements it in a second classroom, while group members again look on. The group will come together again to discuss the observed instruction. Finally, the teachers produce a report of what their study lessons have taught them, particularly with respect to their research question. This approach to professional development fits perfectly with the ideas behind RME and problem posing as it enables communication and interaction among participants and taking responsibility over presented activities, and it is connected to the participants’ realities. Lewis and Hurd (2011) stated, “participants in a lesson study group seek out answers from another, from outside specialists and research and from …teachers’ collective knowledge” (p. 7).
At this point, participants will have all the tools to start working on their own literacy social math problems. They will be challenged with a number of RME problems they will have to modify into social justice problems similar to the problem presented in Session 2. Participants will work in groups and later they will present their problems and their answers to the questions below. Other groups will have the opportunity to give their input on the problems so. This approach will allow participants to go back to their sites with problems that work in a specific grade level and that can be used to create awareness in their students. The professional development “provides an ongoing method to improve instruction, ensuring that good ideas are not just talked about but brought to life for shared observation and analysis” (Lewis & Hurd, 2011, p. 3).

Participants will engage in their groups to modify the RME problems listed below. I recommend solving the problems before finding themes to modify them into social justice problems. This order is important so participants can have a clear understanding of the mathematics behind each one of these problems. Participants will have access to the format with Fosnot’s ideas plus a theme section as shown below, which will help them organize each problem.

Assignment to groups:

1. Work with the RME problems identifying strategies, big ideas, and models.
2. Find a generative theme that could fit the RME problem.
3. Modify them into social justice mathematics.
4. Think about these questions:
What is the purpose of this problem regarding social justice issue? How does it help students to start becoming aware of social disparities?

Can you expand more to move students toward their awakening?

What kind of dialogue can you have in the classroom with this topic?

Further questions from this problem?

**Example RME problem.**

*Hot Tub*

*Marie and Clyde have put a cylindrical hot tub in the center of their orchard. The radius of the circular base is 5 feet, and the tub is 3.5 feet deep.*

a. Clyde has made a canvas cover for the hot tub to keep the leaves out. The cover just fits across the top. How many square feet of material did he need for the cover? (Assume that no material was wasted.)

b. When the hot tub is completely filled, how many cubic feet of water will it hold?

c. Marie has decided to apply a sealant to protect the outside vertical surface of the hot tub against weathering. A quart of sealant will cover 100 square feet of surface. How many quarts will she use?
Example with social justice.

The picture below shows an illegal dumpsite in an underprivileged community. The radius of the circular base is 5 feet, and the tub is 3.5 feet deep.

At the local high school, math students work on a proposal letting city authorities know the amount of dirt they would need to fill the illegal dumpsite and the surface area that would need to be covered with grass.

![Diagram of a cylindrical dumpsite]

Sharing and Discussions

Here is where participants will have the opportunity to share and critique their social justice problems. The idea is to make sure they work for the grade level selected and will provide ample space for dialogue. When returning to their sites, teachers will have a theoretical framework for RME and social justice background provided by Freire’s problem posing education enabling them to keep developing problems on their own. Having access to other participants who belong to their sites will give them the opportunity to discuss and observe what they are accomplishing after the professional development. They will also have access to a blog or wiki-environment where they will be able to find more ideas or share their own. Thus strengthening their skills with RME and social justice math problems.
Summary

Through the professional development’s three sessions, attendees have the opportunity to not only experience Realistic Mathematics Education (RME) and critical pedagogy, but they also have the opportunity to explore and learn about the positive effects from working collaboratively. Attendees will have the time to reflect on strategies for solving open-ended math problems as well as the time to dialogue about many social issues students face in their daily lives. More importantly, participants will have the opportunity to walk through a path that showed them the role of mathematics goes beyond memorizing rules and procedures; mathematics can be used as a tool to achieve full participation in a democratic society; mathematics is a process that can be overwhelming to students “if [they are] not given the tools to be engaged in transforming injustice and alienating if not provided with meaningful opportunities to engage in transformational work” (Berta-Ávila, Tijerina, & Lopez, 2011, p. 80).

I believe teachers play a major role in the education of every student and especially of those who have been marginalized; thus, providing instances where teachers can “air, test, and realign their beliefs” (Lewis & Hurd, 2011, p. 27) cannot be restricted. Professional development with a sense of collaboration, participation, respect, and discourse reassures participants, especially teachers, that they are not alone and solely accountable for students’ successes or failures in their educational journeys. For example, the success of Railside School (Boaler & Staples, 2008) was partly founded on the strong commitment of the mathematics department to collaborate “spending vast
amounts of time designing curricula, discussing teaching decisions and actions, and
generally improving their practice through the sharing of ideas’’ (p. 626). A key
component to achieve equitable education is the value of collaboration that will
eventually be integrated in the students’ activities as they solve open-ended problems in
groups and learn by experience that a less individualist society can promote tolerance,
social change, and access to mobility. Because, as “theorists argued, in a variety of ways
the problems associated with schooling are deeply tied to the reproduction of a system of
social relations that perpetuate the existing structures of domination and exploitation”
(Darder et al., 2003, p. 4). Therefore, educational change starts with cooperation and
teachers working collaboratively help to improve the prospects of the current educational
system that fails “to capture the minds and hearts of their students” (Duncan-Andrade &
Morrell, 2008, p. 71) and opposes to “the concept of public education as a school of
freedom” (Darder et al., 2003, p. 121).
Chapter 4

THE SOCIAL MATH LITERACY PROJECT

By Cecilia V. Lopez

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Morning session 1: A. Introduction – Presentations

Are Open-ended Mathematics Problems Scary?

What Does Math Have to Do with Real Life?

Granted, in each moment, the needs of life are greater than the available means of satisfying them, and therefore the pleasure of life is compromised. Yet this in no way diminishes the pleasure in life that is actually present. Wherever desire finds satisfaction, there is a corresponding quantity of enjoyment—even if there exists, in this creature or others, a huge number of unsatisfied drives. What is diminished is the value of the enjoyment of life. If only a portion of the needs of a living creature find satisfaction, the creature has a corresponding degree of enjoyment.
The smaller the enjoyment is in proportion to the total demands of life in the sphere of the desires in question, the less value that enjoyment will have. We can imagine the value represented by a fraction whose numerator is the enjoyment actually present and whose denominator is the total sum of the needs. When the numerator and the denominator are equal, that is, when all needs are satisfied, then the fraction has a value of one. It becomes greater than one when more pleasure is present in a living creature than its desires demand; it is smaller if the quantity of enjoyment lags behind the sum of desires. But as long as the numerator (the enjoyment) has even the slightest value, the fraction can never equal zero. (Steiner, pp. 210-211)

**B. Working in Groups**

Grouping teachers by schools during the professional development enables teachers to keep working together, sharing their successes and observing each other to improve what needs to be improved. Use ideas of complex instruction to have teachers work together on a Realistic Mathematic Education (RME) problem.

Each group has been assigned, and each person in each group has been assigned a role.
Activity

Each circle represents a member of the group. Complete the graph below by finding two different variables that will relate the four of you.

*Figure 5. Grouping activity.*
**Brainstorming Question in Groups**

*Why is complex instruction necessary to promote equity and access to all in a mathematics classroom?*

1. It promotes classrooms that are multidimensional

   Teachers created multidimensional classes by valuing many dimensions of mathematical work. This was achieved, in part, by implementing open problems that students could solve in different ways. The teachers valued different methods and solution paths and this enabled more students to contribute ideas and feel valued. (Boaler & Staples, 2008, p. 629)

2. Roles

   “These roles, and students’ engagement with mathematics that was supported by them, contributed to a classroom environment in which everyone had something important to do and all students learned to rely upon each other” (Boaler & Staples, 2008, p. 632).

3. Assigning Competence

   “Good Ivan, that is important.” Later when the girls offered a response to one of the teacher’s questions the teacher said, “Oh that is like Ivan’s idea, you’re building on that.” The teacher raised the status of Ivan’s contribution, which would almost certainly have been lost without such an intervention. Ivan visibly straightened up and leaned forward as the teacher reminded the girls of his idea. (Boaler & Staples, 2008, p. 633)
4. Teaching Students are Responsible for Each Other’s Learning

The students in the extract above make the explicit link between teachers asking any group member to answer a question, and being responsible for their group members. They also communicated an interesting social orientation that became instantiated through the mathematics approach, saying that the purpose in knowing individually was not to be better than others but so “you can help others in your group”. There was an important interplay between individual and group accountability in the Railside classrooms. (Boaler & Staples, 2008, p. 635)

**Handout:** Complex Instruction (see Appendix A).

**Handout:** Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School (see Appendix E).

**C. The Problem**

RME problem is introduced.

*A group of friends rent a camping farm for a weekend. A week before the trip, 4 friends had to withdraw from the trip. The remaining participants will therefore pay $5 per person more. Then at the beginning of the weekend, two more friends cannot make the trip and let the group know they will not pay.*
As a result, the remaining friends have to pay $3 extra each.

How many friends eventually went to the camping farm and how much money did they pay?

I will model strategy called the 3Rs. Three people have to read the problem, one at a time, before the group starts working on it. Allow time for the groups to discuss and solve the problem.

D. Check

Check different participants’ approaches to solving the problem and have these groups share their strategies and models with the class. A possible approach to finding the solution to the problem is geometric representation.
Figure 6. Geometric representation.

Solved via this method by Dr. Scott Farrand.

In this picture, horizontal distances indicate the number of people and vertical distances are amounts of money. For example, the rectangle at the bottom whose height is D and whose width is F+6 is the total amount planned to be paid by all the campers before anyone dropped out. The width of that rectangle F+6 represents the total number of people planning on going camping, and the height D represents the amount they thought they would each pay. Thus the area of that rectangle represents the amount the group was going to have to pay for their camping.

When the number of campers was reduced by four, the amount each of the remaining campers would pay increased by 5, so the total payment is now left for the F+2 campers remaining. In this way, the green rectangle on the right is the money that won’t be coming from those four friends, and the fact that it would increase the costs for the others by $5 is represented by the fact that the green rectangle above has the same area.

Notice that this says that 4D=5(F+2) algebraically. You can also just think of those two rectangles as being the same area, and since one is 5/4 the length in one direction, it must be shorter by a factor of 4/5 in the other direction. So, we have that the width of the white rectangle at the bottom (F+2) must be 4/5 of its height D.
Figure 7. Solving the problem.

Notice the blue rectangle on the right represents the money not paid by all the people who backed out of the trip, and the blue rectangle at the top represents the money paid by the final number of campers $F$. The two rectangles have equal areas. The blue rectangle at the top has a dimension of 8 and the one on the right has a dimension of 6. Since the one at the top is 4/3 longer in one direction, it must be ¾ as long in the other direction, and so the white rectangle has a width that is ¾ of the height of the rectangle on the right, so $F = (3/4)D$. Now just focusing on the horizontal direction, at the bottom of the picture, we have the following:
From this, we have that the distance 2 is equal to $(4/5)D − (3/4)D = (1/20)D$.

This means that $D$ must be 40, and the final number of campers $F$ is $¾$ of this, or 30. So there were a total of 36 friends to start with, and they had planned on spending 40 dollars each, but in the end there were 30 campers, and they paid 48 dollars each.

A possible algebraic solution to the problem is Vertical mathematizing.

1. There are $x$ number of friends originally going on the trip. Each will have to pay a total of $y$ dollars. Therefore, these friends together will pay $xy$ dollars.

2. If there are 4 friends that drop out, then there will be $4y$ less money for the trip. Then, $(x - 4)$ will be the number of friends that will pay $4y$ extra dollars in total.

3. Since each remainder friend has to pay $5$ extra, the remaining friends will pay $5(x - 4)$ dollars in total. Then, $4y = 5(x - 4)$

4. Since 2 more friends cannot go, then the remaining friends will have 2 $(5y)$ less money for the trip (or money that needs to be shared by the remaining friends).

Now, the remaining friends represented by $(x - 6)$ will have to add another $3$ to the original amount.

5. At this point, the reminding friends will have to pay an extra amount of $2(y + 5)$ dollars. Therefore, $2(y + 5) = 3(x - 6)$.

6. Using the system of equations $4y = 5(x - 4)$ and $2(y + 5) = 3(x - 6)$
7. \( x = 36 \), There were originally 36 friends going on the trip.

8. Each friend would have originally paid $40

9. They all would have paid $40 \times 36 = $1440

10. There are 36 friends finally going

11. Each will pay $48

**Transition from Participants’ Experiences Solving the Problem to RME’s Principles**

**Brainstorming questions to groups:**

- What was your first approach to solving the problem?
- What was your experience when solving the problem?
- Did you use models to solve the problem?
- Does this approach fit with: teaching pure mathematics and afterwards showing how to apply it?
- Could you teach content that appears in the problem and has been covered in class?

Share reflections with the whole group. Guide group to the following list:

**RME Problems’ Principles**

1. It is not about this one problem, it is about mathematics
2. Promotes class discussions

3. Present open assignments

4. Encourage complex problems

5. Teachers try to understand the students’ reasoning

6. There must be ample opportunities to try things and think about the result

7. Keep the big picture in view

8. Provide new opportunities: return to fundamental issues

9. Establish relationships with previous lessons

At this point, participants will participate in group discussions to generate the key principles of RME through the following questions.

**Brainstorming questions to groups:**

- What is the key component of RME?
- Does RME allow students to solve problems using different approaches?
- Inductive or deductive?
- What is the role of students in RME?
- What is the role of teachers in RME?

After participants have the opportunity to discuss these questions, each group has the opportunity to share their process of thinking. The goal is to achieve the principles of
RME as closely as possible. Participants will get a handout that outlines these principles (see Appendix F).

I. Activity Principle

II. Reality Principle

III. 'Mathematization' Principle

IV. Use of effective models Principle

V. Guidance Principle

VI. Interaction Principle

Presenter will also share Figure 7 with the group as it highlights the importance of math discourse in math classes.

Figure 7. The relationship between knowledge and understanding (CBC, 2012)
Handouts

RME key principles (see Appendix F). Eade (as cited in University of South Dakota, n.d.) outlined the key principles of RME. Prepared by presenter from:

http://people.usd.edu/~kreins/learningModules/RealisticMathematicsEducation.htm

Mathematics education in the Netherlands: A guided tour 1 (see Appendix G), Marja van den Heuvel-Panhuizen Freudenthal Institute, Utrecht University, the Netherlands.

Hans Freudenthal: A Mathematician on Didactics and Curriculum Theory

Afternoon Session 1

Unit Overview

The goal of the afternoon session is to give participants the opportunity to identify, in the problem solved in the morning session, the big ideas covered, the strategies, and the models the students will use to solve a problem.

A group of friends rent a camping farm for a weekend. A week before the trip, 4 friends had to withdraw from the trip.

The remaining participants will therefore pay $5 per person more.

Then at the beginning of the weekend, two more friends could not make the trip and let the group know they would not pay.

As a result, the remaining friends have to pay $3 extra each.

How many friends were there and eventually how much money did they pay?
The ideas of Fosnot on teaching and learning mathematics using RME focuses on children engaging in mathematizing; therefore, she and the developers of *Context for Learning Mathematics* argued, “our work is driven by the desire to transform classrooms into communities of mathematicians: places where children explore interesting problems and craft solutions, justifications, and proofs of their own making” (p. 4). They support the idea that non-linear learning should be composed of three domains: strategies, big ideas, and models.

**Brainstorming**

Using handouts and CCSS, participants will complete Table 4 in their groups.

1. What are *strategies*? Examples?

2. What do you think about the meaning of *big ideas*?

2. What *models* would students use?
### Table 4

**Strategies for Solving Traditional Problems**

<table>
<thead>
<tr>
<th>RME Problem</th>
<th>Big Ideas</th>
<th>Strategies</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>– Create equations that describe numbers or relationships</td>
<td>– Solve equations in one variable&lt;br&gt;– Rearrange variables to highlight a quantity of interest&lt;br&gt;– Solve systems of linear equations</td>
<td>– Table</td>
</tr>
</tbody>
</table>


**Handout.** *Contexts from Learning Mathematics* that shows the use of these three concepts in a math unit (see Appendix B).

**Handout.** CCSS

**Session 2: Learning Mathematics as a Tool to Become Quantitative Literate**
Getting Ready: Review RME Main Ideas

Brainstorming questions to groups.

1. How should math be taught in schools?
   Mathematics as an inductive activity thinking that not all the students will become mathematicians.

2. For what purpose?
   Teach mathematics that students will use to solve problems in everyday life situations

3. To whom?
   Mathematics for all. All students from different ability levels should stay in the same classroom and have the same common curriculum. Heterogeneous groups.

Finding the relationship between Maslow: Hierarchy of Needs, Gardner’s Theory of Multiple Intelligences, and Freudenthal’s ideas
Brainstorming questions to groups.

- Why is it important to be aware of Maslow’s Hierarchy of Needs where you teach?
- Do the different intelligences behind Gardner’s theory play a role in math classrooms?
- What is the relationship between these two theories and what you have learned about Freudenthal’s RME?
- Make a diagram that shows the relationship you see among these three theories.

Share out their thinking process.

The Bridge: From Freudenthal’s Ideas to Freire’s Ideas: Moving Toward Social Justice Mathematics

Brainstorming in groups.

a. What do you think you know about problem-posing education?

b. Is it possible to implement problem-posing education in your mathematics lessons? How?

c. What are generative themes?

d. Could you think of themes that you could use in your math classes?
e. Do you feel comfortable discussing these themes with your students? If not, could you think of topics that you feel comfortable enough as a starting point to a problem-posing approach?

Share reflections and answers to questions.

How and where may Freire’s ideas be included in your diagram?

Share reflections and answers to questions.

**Introducing Freire’s ideas to participants.**

Table 5

*Comparison of Freudenthal and Freire Theories of Education*

<table>
<thead>
<tr>
<th>H. Freudenthal</th>
<th>P. Freire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics is a human activity</td>
<td>Education should be the practice of freedom</td>
</tr>
<tr>
<td>Against deductive approach to teaching math</td>
<td>Against banking education</td>
</tr>
<tr>
<td>Use of real life problems</td>
<td>Use of generative themes</td>
</tr>
<tr>
<td>Mathematics as an activity</td>
<td>Problem posing education</td>
</tr>
<tr>
<td>Mathematical discussions</td>
<td>Dialogue</td>
</tr>
<tr>
<td>Group work- Collaboration</td>
<td>Group work- Collaboration</td>
</tr>
</tbody>
</table>
Table 5 (continued)

<table>
<thead>
<tr>
<th>H. Freudenthal</th>
<th>P. Freire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics is a human activity</td>
<td>Education should be the practice of freedom</td>
</tr>
<tr>
<td>Teacher as a facilitator</td>
<td>Teacher as a facilitator</td>
</tr>
<tr>
<td>Students have an active role in their learning process</td>
<td>Students are not docile listeners</td>
</tr>
<tr>
<td>Applied and pure mathematics are including in real life problems</td>
<td></td>
</tr>
</tbody>
</table>

**The RME problem.**

A group of friends rent a camping farm for a weekend. A week before the trip, 4 friends had to withdraw from the trip.

The remaining participants will therefore pay $5 per person more.

Then at the beginning of the weekend, two more friends cannot make the trip and let the group know they will not pay.

As a result, the remaining friends have to pay $3 extra each.

How many friends eventually went to the camping farm and how much money did they pay?

**Modifying the RME problem to a problem with a social justice theme.**

A company’s tight budget calls for a project to be finished in a set number of hours. The company hires temporary workers to complete the assignment. Before they begin the job,
the manager fires 4 workers and 5 more hours are added to each worker left. Just before they start the job, the manager fires 2 more workers and 3 more hours are added to each of the remaining workers’ hours.

How many extra hours will each worker work?

How many hours are assigned to finish the project?

Reflections.

• What is the theme?

• What is the purpose of this problem?

• Can you expand more to move students towards their conscientization, or conscientização?

• What kind of dialogue can you have in the classroom with this topic?

• Further questions from this problem?

According to a non-profit organization, a group of families from an impoverished community is in need of help to renovate their homes. A group of volunteers signed up for this task that has to be completed in a certain number of hours. A week before the renovation, 4 people canceled and therefore, 5 more hours had to be added to the
remaindering volunteers. Unfortunately, a couple of days before the date, 2 more people canceled and this forced the organizer to add 3 more hours to each volunteer left.

How many volunteers signed up originally?

How many hours do they need to work in total?

Questions to groups.

• What is the theme?

• What is the purpose of this problem?

• Can you expand more to move students toward their conscientization, or conscientização?

• What kind of dialogue can you have in the classroom with this topic?

• Further questions from this problem?

More examples.

Debris from the earthquake-Tsunami in Japan has been found on the Pacific coast shores. This debris can be devastating for the Pacific Ocean wildlife since it can transport invasive species from one continent to another. Cleanup has to be conducted quickly in a certain number of hours. A group of friends from a local school volunteered to help with the cleaning efforts. A couple of days before the cleaning, 4 friends canceled and 5 more hours had to be added to each remaining volunteer. Then, before the set date, 2 more volunteers canceled and therefore, 3 more hours had to be added to the volunteers left.

How many volunteers signed up originally?
How many hours do they need to work in total?

Reflections.

- What is the theme?
- What is the purpose of this problem?
- Can you expand more to move students toward their conscientization, or conscientização?
- What kind of dialogue can you have in the classroom with this topic?
- Further questions from this problem?

Table 6

Example Template to Organize Social Justice Problems

<table>
<thead>
<tr>
<th>Path to Literacy and Social Justice Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Big Ideas</strong></td>
</tr>
<tr>
<td>● Create equations that describe numbers or relationships</td>
</tr>
<tr>
<td><strong>Strategies</strong></td>
</tr>
<tr>
<td>● Solve equations in one variable</td>
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</tr>
<tr>
<td><strong>Models</strong></td>
</tr>
<tr>
<td>● Table</td>
</tr>
<tr>
<td><strong>Social Justice Themes</strong></td>
</tr>
<tr>
<td>● Temporary workers</td>
</tr>
<tr>
<td>● Hiring benefits for temporary workers</td>
</tr>
<tr>
<td>● Benefits for employers when hiring temporary workers</td>
</tr>
<tr>
<td>● Health benefits for workers?</td>
</tr>
<tr>
<td>● Wages</td>
</tr>
<tr>
<td>● Retirement plans?</td>
</tr>
</tbody>
</table>
Session 3: Path to Literacy and Social Justice Mathematics

Brainstorming

1. Reflect on ideas of Realistic Mathematics Education

What are the three main principles in RME that can be applied into literacy and social justice mathematics?
2. Reflect on social justice mathematics

How social justice mathematics can help students to become active participants of society.

3. Reflect of Fosnot’s teaching and learning philosophy and her unit overview to organize presented problems.

4. What are the three main ideas of Fosnot’s unit organizer: big ideas, strategies and the models that the students could use to solve the problems?

Collaborative Lesson Planning

Unit overview. The goal of session 3 is to give participants the opportunity to explore, through the problem solved in the morning session, the big ideas covered, the strategies, and the models the students will use to solve a problem.

A. Lesson study.

Assignment.

1. Work with the RME problems identifying strategies, big ideas, and models.

2. Find a generative theme that could fit the RME problem.

3. Modify them into social justice mathematics.

4. Think about these questions:

   - What is the purpose of this problem?
Can you expand more to move students towards their awakening?

What kind of dialogue can you have in the classroom with this topic?

Further questions from this problem?

B. List of RME problems to participants to work in their groups.

1. In the summer market I bought for 12 books for $12.

   Some cost $0.50, some $1.50 and the rest cost $2.

   How many books did I buy in each price range?

   Example of it in social justice format.

   A college student paid a $1,250 loan that included books and interest. The books cost $750 more than the amount paid for interest. How much did the student pay in interest?

2. In a vase are just as many red, yellow, and blue marbles.

   Half the red marbles have a white dot. Of the yellow marbles, a third of them have a white dot and a quarter of blue marbles have a white dot.

   What percentage of the marbles has a white spot?

   Example of it in social justice format.

   Drinking water in the San Joaquin Valley is polluted with nitrate an inorganic pollutant that can come from fertilizers or decayed organic materials, chloride, and heavy metals such as copper and lead. Local authorities want to assess the issue and for this so they test water in three different areas of one city. They select a sample with the same number of homes in each of the areas A, B, and C.
These are their results:

1/3 of the homes’ water in area A is polluted.
½ of the homes’ water in area B is polluted.
¼ of the homes’ water in area C is polluted.

What percentage of the water is polluted with nitrate in this sample?

Amnesty International wants to learn whether access to education has increased for girls in remote villages in the Middle East. They select a sample with the same number of students in three different schools. These are their results:

1/3 of the students in the first school are girls.
½ of the students in the second school are girls.
¼ of the students in the third school are girls.

What percentage of girls has access to education in remote villages in a middle-east country?

3. A car makes a trip of 2,000 km.

The five tires (including spare) are new. At the end of the ride, the tires are equally worn. How many km did each tire travel at the end? What are the least possible permutations of the tires?

Example of it in social justice format.
A manufacturer is granted 2,000 hours to finish a project. In order to complete this project on time, management has to have their five shifts working full-time. Only four shifts can work equal amounts of hours each time. How many hours can each shift work with the least possible number of permutations? How many 40-hour days would they need to work?

4. In the summer market, I bought 12 books for $12.

Some cost $0.50, some cost $1.50 dollars and the rest cost $2.

How many books did I buy of each kind?

**Example of it in social justice format.**

Melissa runs a program for homeless people in which she lends them clothing for job interviews. Melissa buys gently used clothing for the wardrobe’s program. Last week, she bought 12 pieces of clothing and paid $12. She bought some pieces for $0.50, some for $1.50, and the rest for $2. How many pieces of clothing did she buy at each price?

5. Hot Tub

*Marie and Clyde have put a cylindrical hot tub in the center of their orchard. The radius of the circular base is 5 feet, and the tub is 3.5 feet deep.*
a. Clyde made a canvas cover for the hot tub to keep the leaves out. The cover just fits across the top. How many square feet of material did he need for the cover? (Assume that no material was wasted.)

b. When the hot tub is completely filled, how many cubic feet of water will it hold?

c. Marie has decided to apply a sealant to protect the outside vertical surface of the hot tub against weathering. A quart of sealant will cover 100 square feet of surface. How many quarts will she use?

**Example of it in social justice format.**

The picture below shows an illegal dumpsite in an underprivileged community. The radius of the circular base is 5 feet, and the tub is 3.5 feet deep.

At the local high school, math students work on a proposal letting city authorities know the amount of dirt they would need to fill the illegal dumpsite and the surface area that would need to be covered with grass.
Example of it in social justice format.

Water resources are scarce in some underdeveloped countries. Communities have access to water through dug wells. The picture below shows a dug well and its measurements in a little village in India.

How much water could this well hold? The villagers want to cover the well so it stays clean and is safe for the children. How much material do they need?

C. Sharing group ideas and discussions.

Where to find RME information on the web:
http://fius.org/

Where to find RME problems on the web:

http://www.rekenbeter.nl/EerdereSommen.aspx

Where to find themes on the web:

http://www.radicalmath.org/

Conference

http://creatingbalanceconference.org/

List of handouts:

1. Complex Instruction

2. Creating Mathematical Futures Through an Equitable Teaching Approach: The Case of Railside School

3. Maslow: Hierarchy of Needs

4. H. Freudenthal RME

5. Fosnot: Sample from Context mathematics

6. Freire

Recommended literature:


Chapter 5
DISCUSSION AND RECOMMENDATIONS

Discussion
This researcher believes everyone can succeed in learning mathematics with teachers providing strategies to ensure success and respecting each student’s level of competence. That is, teachers need to have a clear understanding that not every student will become a mathematician, that mathematics is a tool to understand how the world functions, that providing social math interaction is essential, and that creating a healthy and safe environment in which students can develop a sense of acceptance and respect of their contributions and to their learning process are key elements to ensure equitable education. Put differently, it is about teachers believing in the students’ mathematical capacities and helping them flourish; it is about having high expectations, and it is about providing students with more positive experiences. I believe such positive experiences will eventually enable students to actively participate in society, as they will become aware of the importance of having an informed voice, a strong sense of understanding mathematics, and a strong desire to take action to change and reshape a society.

A key role of The Social Math Literacy Project is to not only provide the math content that is the foundation of real-life math problems, but also to provide an understanding that social justice conversations can occur in any class, and why not in math? An important goal of this professional development is to succeed in creating an environment where, first, the participants will become curious and excited about the fact
that RME can provide an environment in which participation and dialogue are the centerpiece to ensure deep understanding of mathematics, and, second, where the participants will acquire the tools to engage in social justice mathematical conversations. This approach to education needs especial attention from teachers and authorities in the form of professional development or working in collaboration to target a teacher’s belief about social issues. It also need to address teachers’ math understanding and the changing of teaching approaches supported by the CCSS for mathematics, a long waited reform to teaching and learning mathematics.

It has been argued and researched that a reform-oriented approach to teaching and learning mathematics enables students to not only achieve higher mathematics levels, but also to acquire understanding of the applicability of mathematics in the real world. Mathematics can be powerful when used at its full potential. Teachers tend to teach their students “the result of an activity rather than teaching the activity itself” (Gravemeijer & Terwel, p. 780), and when this teaching approach is so engraved in society, mathematics turns into a subject in which the “tell me how to solve it” so “I can get to the result” is more important than providing a safe and inviting classroom environment where students can reason about a problem and discuss whether their answers make sense. Therefore, teachers and students forget success is not measured by a single result but by the many different approaches used to get to that result. No doubt that with a more realistic approach to teaching and learning mathematics, with the addition of a social justice component, and with the implementation of the CCSS for mathematics, students can start
experiencing mathematics as a “human activity” (Freudenthal, 1973) and as the “practice of freedom” (Freire, 1993, p. 80).

The Social Math Literacy Project will allow teachers and general participants to learn about the importance of challenging their students in a manner that becomes a long-lasting learning process through the use of open-ended situations modified to have a social justices focus. The emphasis on social justice issues comes at a time when the country, and perhaps many other societies, needs to rethink and restructure accessibility to equitable educational and social mobility, thus enabling students to “see the world not as a static reality, but as a reality in process, in transformation” (Freire, 1993, p. 83).

That is, encouraging dialogue and opportunities to introduce realistic math problems is the main focus, as they give students and teachers the opportunity to discover and unveil oppressive situations as well as to become math competent-literate.

Realistic mathematics education ensures the acquisition of quantitative literacy that develops “confidence with mathematics, the ability to interpret data, to make decisions thoughtfully, and to make use of mathematics in context” (Schoenfeld, 2001, p. 51). Above all, posing social justice problems to students not only develops academic math skills, but also the ability to question, reason, and make sense of numbers in real life and to, ultimately, become critical thinkers. Reasoning “involves drawing logical conclusions based on evidence or stated assumptions” (Strutchens & Quander, 2011, p. 1) and making sense in mathematics “may be considered as developing understanding of a situation, context, or concept by connecting it with existing knowledge or previous
experience” (Strutchens & Quander, 2011, p.1). The above two concepts have been around in the math world for many years, as they are “simultaneously the purpose for learning mathematics and the most effective means of learning it” (Strutchens & Quander, 2011, p. 1). As early as 1819, educator Samuel Goodrich, author of The Child’s Arithmetic, argued, “teaching arithmetic by rote actually prevented children from understanding arithmetic and that they should discover rules by manipulating tangible objects” (UCSB Department of Mathematics Center for Mathematical Inquiry, n.d., para. 3). The NCTM in 2000 advocated building new guidelines for teaching and learning the subject with a strong emphasis on the importance of learning for understanding. In other words, as argued by Perkins (1993), the goal is to prepare students for “further learning and more effective functioning in their lives” (p. 2). To ensure learning for understanding, it is necessary to use teaching strategies and a curriculum that presents the subject through open-ended, structured, or semi-structured problems. Strutchens and Quander (2011) observed a math department that “collectively defined group-worthy problems as having four distinctive properties; they: (a) illustrated important mathematical concepts, (b) included multiple tasks that drew effectively on the collective resources of a student group, (c) allowed for multiple representations, and (d) had several possible solution paths” (p. 107), all of which enable students to reason and make sense.

The Common Core State Standards for mathematics make sense of problems and persevere in solving them, reason abstractly and quantitatively construct viable arguments, model with mathematics, use appropriate tools strategically, attend to
precision, look for and make use of structure, look for and express regularity in repeated reasoning, support the idea that when the subject is presented in context through word problems students are challenged to reason, make sense of quantities and their relationships, check their answers, apply math to everyday situations, and look for patterns or structure all of which prepare them to become mathematically proficient. That is, when students are asked to explain, verbalize their ideas, and communicate the core or the meaning of the math problem in their own words, they then start manipulating symbols and looking for relationships that will ensure learning for understanding rather than learning the “how to solve” meaningless math problems using a set of rules and algorithms given by their teachers. For example, Strutchens and Quander (2011) observed:

Students were not expected to quickly move through pages of problems or to memorize algorithms; instead, these classrooms valued understanding and making sense of mathematical concepts. Students who could solve problems—not by mindless manipulation but through careful and sophisticated reasoning—were held in high esteem. (p. 107)

Other significant research supporting making sense and reasoning in mathematics is the work of Boaler and Staples (2008). They argued that students’ experiences, successes, and academic achievements did not focus on tests scores, but rather on how the subject helped them to learn mathematics and shape them as problem solvers and critical thinkers. Boaler (2011) observed teachers valued multidimensional classrooms,
or “many dimensions of mathematical work” (p. 3). For example, they gave students the opportunity to “work in groups [as well as] giving students ‘group-worthy problems’: open-ended problems that illustrated important mathematical concepts, allowed for multiple representations, and focused on sense making and reasoning” (p. 3). As stated by Boaler (2011), meaningful math problems are key for mathematical success, as these types of problems involve more than just finding the correct answer; they involve “asking and making sense of questions, drawing pictures and graphs, rephrasing problems, and justifying and reasoning” (p. 3). She shared the experience of one lower-achieving student asking the following questions of his group members: “‘Why you did this?’ And then I’d be, like, ‘I don’t get it why you got that.’ And then, like, sometimes the answer’s just, like, they be, like, ‘Yeah, he’s right and you’re wrong.’ But, like, why?” (p. 5).

Asking why from teachers to students as well as among students is very significant in different dimensions because, as Boaler (2011) explained, this student:

had learned that it was his right to ask why and to keep asking why until he understood, which led him to encourage the student from whom he was copying to reason and justify his thinking, giving Juan more access to understanding. (p. 5)

Thus, the role of teachers revolves around providing their students with instances in which they can be challenged, surprised, and engaged in different mathematical tasks, ensuring that language development occurs, that different approaches to solving a problem are respected, and that no student is left behind in the classrooms. Moschkovich
(2011) argued, “learning to communicate mathematically is not merely or primarily a matter of learning vocabulary. During discussions in mathematics classrooms, students are also learning to describe patterns, generalize, and use representations to support their claims” (p. 19). That is, students are developing a deep understanding of math concepts through their ability to convey process, reasoning, and respectful criticism of other students’ work and to use tables and graphs; consequently, they learn mathematics for life.

However, as well stated by Boaler (2011), “moving from a procedural to a sense-making approach in mathematics is not straightforward for teachers; it takes commitment, training, and time” (p. 15). It was argued in Chapter 2, through the work of Shulman (1986), Kajander (2010), and Ingersoll (2003) that good teaching requires teachers’ pedagogical content preparation and knowing the subject well in order to deliver the subject in depth so students can achieve math proficiency. Thus, how do we teach the CCSS-Math with such a small number of qualified teachers? How do we make sure students will reach academic content-related discourse in their math classes? How do we make sure English Learners are not left to fall between the cracks in mathematics learning? How do we guarantee equitable math education for all students? These are not easy questions to answer; however, as stated by Strutchens and Quander (2011), there is a need to understand that “all students need access each year to a coherent, challenging mathematics curriculum taught by competent and well-supported mathematics teachers” (p. 117).
Many states recognize the urgency in that professional development is key to the success of the implementation of the new standards. For example, the state of Georgia is working with Georgia Public to create and disseminate CCSS Professional Learning videos, New Mexico developed an understanding of the statewide transition plan and began creating their own strategies for delivery of content and support, and Oklahoma uses a train-the-trainer models in which small groups learn content and strategies and then share the material with the teachers in their schools (National Governors Association, 2010). However, training teachers should go beyond informational experiences about the new CCSS. Teachers will need intensive professional development and long-term experiences, as they will need to change their approach to teaching the subject.

Professional development needs to engage them in thinking deeply about the practices that they use and the impact of these practices on their students, to explore new approaches that may be more effective, and to have classroom support as they begin to implement new methodologies that promise to enhance reasoning and sense making. (Strutchens & Quander, 2011, p. 5)

**Recommendations for Further Studies**

*The Literacy Math Social Project* is professional development with the goal of helping teachers understand two important key components that could be approached hand-in-hand in education and mathematics: social justice issues and the use of real-life content problems. These two components in education today are key as we enter an era
of change regarding the standards for mathematics that will include not only changes in classroom teaching strategies, but also changes in textbooks and assessment. A critical piece that will contribute to make the most difference in the implementation of this new approach to teaching and learning mathematics and the exposure students will gain from this experience are teachers. There is tension among teachers grounded in their beliefs about mathematics as a static subject that needs to be taught traditionally who also express resistance to social issues being brought into classrooms for discussion. How do we achieve the rupture of the educational paradigm existing in today’s math classrooms? How do we achieve that needed change? A recommendation for future study might focus on how to begin engaging teachers in collaborative dialogue that might support a change of the present focus of mathematics to a more critical approach. This study might help to understand there is a sense of urgency to make changes in teaching and learning mathematics, as the achievement gap has continued to deny access to social involvement for underprivileged students.

There is hope in the mathematics community as the new Common Core Standards might enable math education to move away from No Child Left Behind. In other words, the changes the new core standards for mathematics will bring into classrooms around the country will ensure teachers “seek to foster actual change in on-going classroom teaching” (Gravemeijer & Terwell, p. 780) opening doors not only incorporates new educational materials but also teaching approaches that will start engaging students in real life content mathematic problems and eventually achieve equitable education.
Complex Instruction

1. Multidimensional Classrooms

In many mathematics classrooms there is one practice that is valued above all others—that of executing procedures (correctly and quickly). The narrowness by which success is judged means that some students rise to the top of classes, gaining good grades and teacher praise, while others sink to the bottom. In addition, most students know where they are in the hierarchy created. Such classrooms are unidimensional—the dimensions along which success is presented are singular. In contrast, a central tenet of the complex instruction approach is what the authors refer to as multiple ability treatment. This treatment is based upon the idea that expectations of success and failure can be modified by the provision of a more open set of task requirements that value many different abilities. Teachers should explain to students that “no one student will be ‘good on all these abilities’ and that each student will be ‘good on at least one’” (Cohen & Lotan, 1997, p. 78). Cohen and Lotan provide theoretical backing for their multiple ability treatment using the notion of multidimensionality (Rosenholtz & Wilson, 1980; Simpson, 1981).

2. Roles

Students were placed into groups they were given a particular role to play, such as facilitator, team captain, recorder/reporter or resource manager (Cohen & Lotan, 1997). The premise behind this approach is that all students have important work to do in groups, without which the group cannot function.

3. Assigning Competence

An interesting and subtle approach that is recommended within the complex instruction literature is that of assigning competence. This is a practice that involves teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group’s attention; asking a student to present an idea; or publicly praising a student’s work in a whole class setting. Cohen (1994) recommends that if student feedback is to
address status issues, it must be public, intellectual, specific, and relevant to the group task (p. 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work; and the specific dimension means that students know exactly what the teacher is praising. This practice is linked to the multidimensionality of the classroom which values a broad range of work and forms of participation. The practice of assigning competence demonstrated the teachers’ commitment to equity and to the principle of showing what different students could do in a multifaceted mathematical context.

4. Teaching Students to be Responsible for Each Other’s Learning

A major part of the equitable results attained at Railside came from the serious way in which teachers taught students to be responsible for each other’s learning. Many schools employ group work which, by its nature, brings an element of responsibility, but Railside teachers went beyond this to encourage the students to take the responsibility very seriously. In previous research on approaches that employ groupwork, students generally report that they prefer to work in groups and they list different benefits, but the advantages usually relate to their own learning (see Boaler, 2000, 2002a, 2002b).
Field Trips and Fund-Raisers

Introducing Fractions

Catherine Twomey Fosnot
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Unit Overview

The focus of this unit is the development of fractions. It begins with the story of a class field trip. The class split into four groups and each group was given submarine sandwiches to share for lunch. Upon returning from their trip, the students quarreled over whether some received more to eat than others.

Note: This unit begins with the fair sharing of submarine sandwiches on a field trip. This context was field-tested by the Freudenthal Institute and the University of Wisconsin, under the direction of Thomas Romberg and Jan de Lange, in preparation for the writing of Mathematics in Context: Some of the Paris (van Galen, Wijers, Burrill, and Spence 1997) and it has been researched and written about extensively as it is used in this unit by Feuer and Dolk (2002).

This story context sets the stage for a series of investigations in this unit. First, students investigate whether the situation in the story was fair—was the quarreling justified?—thereby exploring the connection between division

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The Landscape of Learning

<table>
<thead>
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<th>BIG IDEAS</th>
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<td>Fractions are relations—the size or amount of the whole matters</td>
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<td>Fractions may represent division with a quotient less than one</td>
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<td>With unit fractions, the greater the denominator, the smaller the piece is</td>
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<td>Pieces don’t have to be congruent to be equivalent</td>
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<td>For equivalence, the ratio must be kept constant</td>
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<td>Using a ratio table as a tool to make equivalent fractions</td>
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<th>MODELS</th>
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<td>Ratio table</td>
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<td>Measurement</td>
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<td>Fraction bars</td>
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<td>Double open number line</td>
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and fractions, as well as ways to compare fractional amounts. As the unit progresses, students explore other cases to determine fair sharing and then make a ratio table to ensure fair sharing during their future field trips. They also design a 60k bike course for a fund-raiser, a context that introduces a bar model for fractions and provides students with another opportunity to explore equivalent fractions.

Several mini-lessons for division of whole numbers using simplified equivalents are also included in the unit. These are structured using strings of related problems as a way to more explicitly guide learners toward computational fluency with whole-number division and to build a connection to equivalent fractions.

**Note:** The context for this unit assumes that your students have had prior experience with arrays for multiplication and division, as well as partitive and quotative division with whole numbers. If this is not the case, you might find it helpful to first use the units *The Teachers’ Lounge* and *Mini-lessons Throughout the Year: Multiplication and Division*.

**The Mathematical Landscape**

Have you ever watched students trying to fold a strip of paper into thirds? Because this is so difficult to do, they often make three equal pieces first and then snip off the sliver of the strip that remains and declare they have made thirds! Of course they have changed the whole, so they do not have 1/3 of the original strip; just three congruent pieces with part of the strip thrown away! Constructing the idea that fractions are relations and thus the size or the amount of the whole matters is an important big idea underlying an understanding of fractions. The conception that removing a small piece doesn’t matter results from fractions being taught as a shading activity of part-whole relations divorced from division. Research by Leon Streetland (1991) of the Presidential Institute in the Netherlands has shown that learners would do better if they started exploring fractions in more realistic fair-sharing contexts, such as one candy bar shared among three people. No child is willing to discard a piece in this context!

---

**BIG IDEAS**

This unit is designed to encourage the development of some of the big ideas underlying fractions:

- **fractions are relations—the size or amount of the whole matters**
- **fractions may represent division**
  (both partitive and quotative forms) with a quotient less than one
- **with unit fractions, the greater the denominator, the smaller the piece is**
- **pieces don’t have to be congruent to be equivalent**
- **for equivalence, the ratio must be kept constant**

- **Fractions are relations—the size or amount of the whole matters**

Fractions are relations: a ratio of part to whole (2 parts out of 4) or a rate (3 sandwiches for 4 people). Fractions can also be operators. For example, 3/4 could actually be more than 1/2 if we are talking about 3/4 of 15 versus 1/2 of 10! Constructing the idea that fractions are relations and that the size or amount of the whole matters is a critical step in understanding fractions.

- **Fractions may represent division**
  (both partitive and quotative forms) with a quotient less than one

Just as there are different ways of thinking about division, there are different ways of thinking about fractions. For example, 12 cookies shared among 3 children is 1/3 = 4. This example is a rate, a partitive form of division: 12 for 3, or 4 for 1. The problem: 12 cookies, 3 to a bag, how many bags? can also be represented as 12/3. But this example requires us to think about measurement. How many times does a group of 3 fit into 12? This is a quotative form of division. Fractions can be thought of similarly. Three submarine sandwiches shared among 4 people (partitive) is 3/4, or 3/4 for 1. In this case, 3/4 can also be thought of as how many times a bar of 4 units fits into a bar of 3 units (quotative). Not once—only 3/4 of the 4 fits.
When fractions are developed with fair-sharing division situations, it is easier for learners to construct the big idea that multiplication and division are related to fractions: 3 subs shared among 5 children results in each child getting \( \frac{3}{5} \) of each sub. Because there are 5 subs, everyone gets \( 3 \times \frac{3}{5} \) or \( \frac{3}{5} \). The idea of fractions as division is an important idea on the landscape. To deeply understand fractions, learners need to generalize the partitive and quotative relations: 1 candy bar shared among 8 children is equivalent to 1 out of 8 parts. Another way to think about this is to think of how 3 subs shared with 5 people (3 divided by 5) results in \( \frac{3}{5} \) of one sub. Learners do not need to know the terms partitive and quotative, but they do need to know that 3 things shared among 5 people results in \( \frac{3}{5} \). The slash symbol between the numerator and denominator is just a symbol to represent division.

- **With unit fractions, the greater the denominator, the smaller the piece is.**

Students initially may think the reverse—that unit fractions with greater denominators represent greater amounts—because they attempt to generalize their knowledge of whole number to fractions. For example, they reason that since 8 is greater than 7, \( \frac{1}{8} \) must be greater than \( \frac{1}{7} \). When students are introduced to fractions in fair-sharing contexts, it is easy for them to understand that the greater the denominator, the smaller the piece. When eight people share a pizza, each piece is smaller than when seven people share it.

- **Pieces don’t have to be congruent to be equivalent.**

Fair-sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. For example, a square can be cut into two triangular halves (diagonally) or two rectangular halves (vertically). The pieces may look different, but the areas are equivalent.

- **For equivalence, the ratio must be kept constant.**

Equivalence of fractions is often a difficult concept for students to understand. Traditionally, learners have been taught to make equivalent fractions by multiplying or dividing by one (in the form of \( \frac{2}{2} \) etc.). Even when learners successfully use this strategy and can parrot back that they are multiplying by one, they may not be convinced that the fractions are really equivalent. Understanding that \( \frac{1}{4} \) is not \( \frac{1}{2} \) doubled requires that learners understand the implicit 2 for 1 ratio. Imagine a rectangle cut into tenths with six out of the ten shaded, as shown below. Establishing equivalence requires that every two become a new part (2 for 1). Then there are fifths (cut with the arrows) instead of tenths and only three shaded parts instead of six.

\[
\begin{array}{c}
\frac{3}{5} = \frac{6}{10}
\end{array}
\]

Fair sharing is a very helpful context for exploring equivalence because it is often easier for students to work with than the part-whole model. For example, it is much easier for learners to understand that 3 subs shared among 5 children is an equivalent situation to 6 subs shared among 10 children. If you double the number of people, you better double the number of subs! This is an example of keeping the ratio constant.

**STRATEGIES**

As you work with the activities in this unit, you will also notice that students will use many strategies to derive answers and to compare fractions. Here are some strategies to notice:

- using landmark unit fractions or using common fractions
- using decimal and/or percentage equivalents
- using a ratio table as a tool to make equivalent fractions
- using multiplication and division to make equivalent fractions
- using a common whole to compare fractions
Using landmark unit fractions or using common fractions

When students are faced with fair-sharing situations, they will usually mathematize them in one of two ways. For example, when sharing subs they may use unit fractions (fractions with numerators of one) and cut a sub with landmark amounts first (such as 1/2 or 1/4) and continue with each sub and then try to add the amounts up (in the history of fractions this strategy is very similar to how early Egyptians thought about fractions). To represent sharing 3 subs among 5 people, students will usually do the following: Cut each sub in half. Everyone gets 1/2. Cut the remaining 1/2 up into fifths. This produces 1/5 of a sub for each person. So everyone gets 1/2 + 1/5.

The second strategy is different. It produces common fractions—in which the numerator is not one. Here each sub is cut into fifths at the start, since there are five people. Now everyone gets 3 × 1/5 or 3/5 of a sub.

Each of these strategies brings learners to different roadblocks. In both cases learners may struggle to determine what the total amount is. When using unit fractions, they must figure out how to add fractions with different denominators, and they must determine what to call 1/2 of 1/5. They are faced with the idea that the whole matters. They need to know what the whole is in order to name the part: is the fifth the whole, or is the sub the whole? Is the silver a fifth or a tenth? When using common fractions, students end up with 3 × 1/5 and are faced with the relationship of division and multiplication: 3 subs divided by 5 people produces 1 divided by 5, times 3. They also encounter the relationship between partitive (fair sharing) and quotative (part-whole) representations of fractions and can be pushed to generalize about these relations.

Using decimal and/or percentage equivalents

Some students may recognize the “3 subs for 5 people” situation as division and use the long division algorithm or grab a calculator. This move will result in a decimal quotient of 0.6. A few students may even turn this into a percentage equivalent and report that everyone gets 60 percent. In these cases, it is important to have them develop a justification for the equivalence using pictures of the sub. Can they show that 0.6, or 60 percent, is equivalent to 3/5? Justifying equivalence in a drawing (as on page 7) may become a roadblock. As you progress through this unit, look for opportunities for rich discussion around these strategies and big ideas and help students work through the roadblocks.

Using a ratio table as a tool to make equivalent fractions

The fair-sharing situations in this unit help to generate use of the ratio table as a tool to make equivalent fractions. Students will notice patterns and develop strategies to make equivalent fractions. The first strategy you will most likely see is a doubling strategy. If you double the amount to be shared and you double the number of people, the result is the same. This strategy is very helpful in some cases, such as 1/4 = 5/20 = 1/20, but it is not sufficient for all cases, for example when numbers are not as friendly. Students can also use the ratio table to keep rates equivalent by adding numerators and adding denominators. Three subs for 4 people and 6 subs for 8 people can be used to derive 9 subs for 12 people.

Using multiplication and division to make equivalent fractions

Eventually students will construct the strategy of using multiplication and division to make equivalent fractions. For example, to find an equivalent for 3/5 with a denominator of 10, students must multiply the 3 by 2, since 10 divided by 5 equals 2, to produce 6, or students can divide the 10 by the 5 to produce 2 and then multiply the 3 by 2 to produce 6. Because fractions can be thought of as division and simplifying fractions is an important strategy for making equivalents, it is often a very efficient strategy to use even when dividing whole numbers. For example, 300 divided by 12 can be simplified to 100 divided by 4. The simplified version can be done mentally. This unit was carefully crafted and field-tested to ensure that simplifying is specifically addressed, both for fractions and for division with whole numbers. Not only will a discussion on this idea come up as students explore fair-sharing situations, but mini-lessons are also included to support the use of this strategy for whole-number division.
Using a common whole to compare fractions

Comparing fractions creates another hurdle for students. How can they make a common whole to compare them? Initially, many numbers that are not common multiples may be tried as denominators. It is important to let students grapple with this problem. Eventually they will gravitate toward the realization that a common whole can be made by finding a common multiple. While a common multiple may be the result of their process, it is not important to name it as a common multiple at this time. By allowing their struggle and supporting their quest to find a common whole, you give students the opportunity to "own" the solution when they finally hit on the idea of common denominators.

Mathematical Modeling

Several mathematical models are developed in this unit, but only two are being introduced for the first time: the ratio table and the measurement model. With the ratio table model, students are supported to envision equivalent fair-sharing relationships. As the unit progresses, the ratio table model is used to support the development of various strategies for finding equivalent rates. Then the measurement model is introduced and equivalent fractions are explored with fraction bars and number lines.

Models go through three stages of development (Gravemeijer 1999; Fosnot and Doll 2002):

- **model of the situation**
- **model of students' strategies**
- **model as a tool for thinking**

Model of the situation

Initially models grow out of modeling the situation—in this unit, the ratio table emerges as a chart to ensure fair sharing during future field trips. The measurement model emerges as students design a 60k bike course.

Model of students' strategies

Students benefit from seeing the teacher model their strategies. Once a model has been introduced as a model of the situation, you can use it to model students' strategies as they explain their thinking. If a student says, "I doubled the numerator and doubled the denominator," draw the following:

```
<table>
<thead>
<tr>
<th>Subs</th>
<th>(\frac{3}{5})</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>People</td>
<td>1 (\times 2)</td>
<td>10</td>
</tr>
</tbody>
</table>
```

If a student says, "I added the 5 and 10 to make 15, so I added the 3 and 6 to make \(\frac{9}{5}\)," draw the following:

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<table>
<thead>
<tr>
<th>Subs</th>
<th>(\frac{3}{5})</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>People</td>
<td>1 (\times 2)</td>
<td>10</td>
</tr>
</tbody>
</table>
```

Representations like these give students a chance to discuss and envision each other's strategies.

Model as a tool for thinking

Eventually learners become able to use the model as a tool to think with—they will be able to use it as a tool to prove and explore their ideas about proportional reasoning. Ratio tables can be presented as t-charts:

<table>
<thead>
<tr>
<th>Number of Subs</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5}{6})</td>
<td>(1)</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

Measurement models become number lines where fractions can be placed as numbers. In time, this model is very helpful for addition and subtraction of fractions. Although operations are not the focus of this unit, you may find some students beginning to explore that topic.
Many opportunities to discuss these landmarks will arise as you work through this unit. Look for moments of puzzlement. Don’t hesitate to let students discuss their ideas and check and recheck their strategies. Celebrate their accomplishments—they are young mathematicians at work! A graphic of the full landscape of learning for fractions, decimals, and percents is provided on page 11. The purpose of the graphic is to allow you to see the longer journey of your students’ mathematical development and to place your work with this unit within the scope of this long-term development. You may also find it helpful to use this graphic as a way to record the progress of individual students for yourself. Each landmark can be shaded in as you find evidence in a student’s work and in what the student says—evidence that a landmark strategy, big idea, or way of modeling has been constructed. In a sense, you will be recording the individual pathways your students take as they develop as young mathematicians!

References and Resources


The landscape of learning: fractions, decimals, and percents on the horizon showing landmark strategies (rectangles), big ideas (circles), and models (triangles).

Unit Overview
Investigating Multiplication and Division

Grades 3–5

Catherine Twomey Fosnot
Our Teaching and Learning Philosophy

Depth versus Breadth

Lester Rubenfeld, a mathematician, once said, “When I’m working on a problem, I like examining it in many ways, going deeper and deeper to really understand the structure of it. It’s like playing in the playground” (Kellison, Fosnot, and Dolk 2004). Our work is driven by the desire to transform classrooms into communities of mathematician places where children explore interesting problems and craft solutions, justifications, and proofs of their own making. Many materials currently on the market do not allow children to examine problems deeply. Breadth is the goal, not depth. We believe, however, that covering fewer topics in more depth will better prepare children for the tests they take and for higher-level courses. And the research supports our belief. In countries where curricula focus on fewer topics each year, students actually score higher on international assessments (Stigler and Hiebert 1999). As we developed these units we pushed for depth, not breadth. We wanted children to have opportunities to “play in the playground” of the problem, rather than merely complete quantities of similar problems or meaningless activities for practice.

The Landscape of Learning

The rich, open investigations we’ve developed allow children to engage in mathematicizing in a variety of ways. We honor children’s initial attempts at mathematicizing, at the same time supporting and challenging children to ensure that important big ideas and strategies are being developed progressively. Our approach should not be confused with what is commonly called “developmentally appropriate practice,” where teachers assess every child, assign a stage, and then match tasks to each child. Our approach emphasizes emergence. Learning, real learning, is messy; it is not linear. We conceive of learning as a developmental journey along a landscape of learning. This landscape is composed of landmarks in three domains: strategies, big ideas, and models.
Strategies, Big Ideas, and Models

Strategies can be observed. They are the organizational schemes children use to solve a problem; for example, they might use repeated addition, skip-count, or double and halve when multiplying, or use partial quotients or simplify first when dividing.

Underlying these strategies are big ideas. Big ideas are "the central, organizing ideas of mathematics—principles that define mathematical order" (Schifter and Fosnot 1993, 35). Big ideas are deeply connected to the structures of mathematics. They are also characteristic of shifts in learners' reasoning—shifts in perspective, in logic, in the mathematical relationships they set up. As such, they are connected to part-whole relations—to the structure of thought in general (Posner 1977). In fact, that is why they are connected to the structures of mathematics. Through the centuries and across cultures as mathematical big ideas developed, the advances were often characterized by paradigmatic shifts in reasoning. That is because these structural shifts in thought characterize the learning process in general. Thus, these ideas are "big" because they are critical ideas in mathematics itself and because they are big leaps in the development of the structure of children's reasoning. Here are some of the big ideas you will see children constructing as they work with the materials in this package: unitizing, multiplication is distributive (the distributive property), and factors can be grouped in a variety of ways and orders (associative and commutative properties).

Finally, mathematizing demands the development of mathematical models. In order to mathematize, children must learn to see, organize, and interpret the world through and with mathematical models. This modeling often begins simply as representations of situations, or problems, by learners. For example, learners may initially represent a situation with connecting cubes or a drawing. These models of situations eventually become generalized as learners explore connections across contexts—for example, using graph paper arrays as blueprints in designing new boxes for Muffles' assortments, determining the number of truffles in various boxes when only the dimensions are known. Generalizing across contexts allows learners to develop more encompassing mental models to think about situations with—for example, the blueprint becomes an open array to explore the use of partial products. At this point, teachers use the emerging model didactically, representing children's invented computation strategies for multiplication and division on an open array. This stage bridges learning from informal solutions specific to a context toward more formal, generalizable solutions—from models of thinking, to models for thinking (Behrmann, Gravemeijer, and van Lieshout 1997; Gravemeijer 1999). Models that are developed well can become powerful tools for thinking.
Learning as Development

Now picture a landscape. The landscape for multiplication and division is on page 16. On the horizon is a deep understanding of these topics. Along the way are many developmental landmarks—strategies, big ideas, and ways of modeling that as a teacher you will want to notice, support the development of, challenge learners to construct, and celebrate. The units in this package are designed to support children on this journey. Each unit has a different focus and zooms in on a section of the landscape. You will find this information on the first page of the overview in each unit.

Teaching mathematics is about facilitating mathematical development. This means that you cannot get all learners to the same landmarks at the same time, in the same way, any more than you can get all toddlers to walk at the same time, in the same way. All you can do is provide a rich environment, turn your classroom into a mathematical community, and support the development of each child in the journey toward the horizon.
The landscape of learning: multiplication and division on the horizon showing landmark strategies (rectangles), big ideas (circles), and models (triangles).
Abraham Maslow developed a theory of personality that has influenced a number of different fields, including education. This wide influence is due in part to the high level of practicality of Maslow's theory. This theory accurately describes many realities of personal experiences. Many people find they can understand what Maslow says. They can recognize some features of their experience or behavior which is true and identifiable but which they have never put into words.

Maslow is a humanistic psychologist. Humanists do not believe that human beings are pushed and pulled by mechanical forces, either of stimuli and reinforcements (behaviorism) or of unconscious instinctual impulses (psychoanalysis). Humanists focus upon potentials. They believe that humans strive for an upper level of capabilities. Humans seek the frontiers of creativity, the highest reaches of consciousness and wisdom. This has been labeled "fully functioning person", "healthy personality", or as Maslow calls this level, "self-actualizing person."

Maslow has set up a hierarchic theory of needs. All of his basic needs are instinctoid, equivalent of instincts in animals. Humans start with a very weak disposition that is then fashioned fully as the person grows. If the environment is right, people will grow straight and beautiful, actualizing the potentials they have inherited. If the environment is not "right" (and mostly it is not) they will not grow tall and straight and beautiful.

Maslow has set up a hierarchy of five levels of basic needs. Beyond these needs, higher levels of needs exist. These include needs for understanding, esthetic appreciation and purely spiritual needs. In the levels of the five basic needs, the person does not feel the second need until the demands of the first have been satisfied, nor the third until the second has been satisfied, and so on. Maslow's basic needs are as follows:

**Physiological Needs**
These are biological needs. They consist of needs for oxygen, food, water, and a relatively constant body temperature. They are the strongest needs because if a
person were deprived of all needs, the physiological ones would come first in the person's search for satisfaction.

**Safety Needs**

When all physiological needs are satisfied and are no longer controlling thoughts and behaviors, the needs for security can become active. Adults have little awareness of their security needs except in times of emergency or periods of disorganization in the social structure (such as widespread rioting). Children often display the signs of insecurity and the need to be safe.

**Needs of Love, Affection and Belongingness**

When the needs for safety and for physiological well-being are satisfied, the next class of needs for love, affection and belongingness can emerge. Maslow states that people seek to overcome feelings of loneliness and alienation. This involves both giving and receiving love, affection and the sense of belonging.

**Needs for Esteem**

When the first three classes of needs are satisfied, the needs for esteem can become dominant. These involve needs for both self-esteem and for the esteem a person gets from others. Humans have a need for a stable, firmly based, high level of self-respect, and respect from others. When these needs are satisfied, the person feels self-confident and valuable as a person in the world. When these needs are frustrated, the person feels inferior, weak, helpless and worthless.

**Needs for Self-Actualization**

When all of the foregoing needs are satisfied, then and only then are the needs for self-actualization activated. Maslow describes self-actualization as a person's need to be and do that which the person was "born to do." "A musician must make music, an artist must paint, and a poet must write." These needs make themselves felt in signs of restlessness. The person feels on edge, tense, lacking something, in short, restless. If a person is hungry, unsafe, not loved or accepted, or lacking self-esteem, it is very easy to know what the person is restless about. It is not always clear what a person wants when there is a need for self-actualization.

The hierarchic theory is often represented as a pyramid, with the larger, lower levels representing the lower needs, and the upper point representing the need for self-actualization. Maslow believes that the only reason that people would not move well in direction of self-actualization is because of hindrances placed in their way by society. He states that education is one of these hindrances. He recommends ways education can switch from its usual person-stunting tactics to person-growing approaches. Maslow states that educators should respond to the potential an individual has for growing into a self-actualizing person of his/her own kind. Ten points that educators should address are listed:

i. We should teach people to be *authentic*, to be aware of their inner selves and to hear their inner-feeling voices.
ii. We should teach people to *transcend their cultural conditioning* and become world citizens.

iii. We should help people *discover their vocation in life*, their calling, fate or destiny. This is especially focused on finding the right career and the right mate.

iv. We should teach people that *life is precious*, that there is joy to be experienced in life, and if people are open to seeing the good and joyous in all kinds of situations, it makes life worth living.

v. We must *accept the person* as he or she is and help the person learn their inner nature. From real knowledge of aptitudes and limitations we can know what to build upon, what potentials are really there.

vi. We must see that the person's *basic needs are satisfied*. This includes safety, belongingness, and esteem needs.

vii. We should *refreshen consciousness*, teaching the person to appreciate beauty and the other good things in nature and in living.

viii. We should teach people that *controls are good*, and complete abandon is bad. It takes control to improve the quality of life in all areas.

ix. We should teach people to transcend the trifling problems and *grapple with the serious problems in life*. These include the problems of injustice, of pain, suffering, and death.

x. We must teach people to be *good choosers*. They must be given practice in making good choices.
APPENDIX D

Critical Pedagogy Handout

Critical mathematics education for the future

Ole Skovsmose
Aalborg University, Denmark

Abstract
Mathematics education can mean disempowerment or empowerment. It does not contain any strong "spine", but could collapse into rigid forms and support problematic features of any social development. However, mathematics education can also contribute to the creation of a critical citizenship and support democratic ideals. The socio-political roles of mathematics are neither fixed nor determined. In this sense I talk about mathematics education as being critical.

I see critical mathematics education as a preoccupation with challenges emerging from the critical nature of mathematics education. Critical mathematics education refers to concerns which have to do with both research and practice, and a concern for equity and social justice being one of them. Here I want to refer to the following challenges: (1) How do processes of globalization and ghettoizing frame mathematics education? (2) What does it mean to go beyond the assumptions of Modernity? (3) How should "mathematics in action", including a mixing of power and mathematics, be interpreted? (4) What forms of suppression can be exercised through mathematics education? (5) How could mathematics education provide empowerment?

Such questions reflect an uncertainty with respect to the possible socio-political functions of mathematics education. Facing this uncertainty, this aporia, is a characteristic of critical mathematics education. This cannot be based on any political or epistemological foundation. Our situation is similar to that of those who need to construct a ship while swimming around in the open sea.

Introduction
Does critical mathematics education embody an obsolete line of thought? Is it just a leftover from an outdated leftist educational movement? If not, what could critical mathematics education mean today and for the future?

I see critical mathematics education as an expression of concerns for what socio-political roles mathematics education might play. Critical mathematics education has many roots, one of which is found in Critical Theory that also nourished critical education in general. Sources of inspiration can, however, also bring about assumptions, which can obstruct further development. I suggest there is a need for critical mathematics education to become re-conceptualised, and developed with new references. Roots are important, but an uprooting can sometimes be necessary.

There is an ongoing discussion in education, especially amongst critical educators, about the relationship between research and practice. One could expect a

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The remainder of the article can be found here: www.educ.fc.ul.pt/docentes/.../CME_for_the_Future.pdf
CRITICAL MATHEMATICS EDUCATION: AN APPLICATION OF PAULO FREIRE’S EPISTEMOLOGY

With a new Afterword (2010)

Marilyn Frankenstein

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ABSTRACT:

Paulo Freire’s critical education theory is “re-invented” in the context of a mathematics curriculum for urban working-class adults. The problems Freire poses for teachers in that context are explored and work of other theorists which deepens or questions aspects of Freire’s theory is discussed. Next, Freire’s theory is applied to teaching basic mathematics and statistics for the social sciences. It is argued that such mathematical literacy is vital in the struggle for liberatory social change in our advanced technological society. Finally, this reflection on practice is used to pose further problems to be explored in the creation and re-creation of the “pedagogy of the oppressed.”

Knowledge of basic mathematics and statistics is an important part of gaining real popular, democratic control over the economic, political, and social structures of our society. Liberatory social change requires an understanding of the technical knowledge that is too often used to obscure economic and social realities. When we develop specific strategies for an emancipatory education, it is vital that we include such mathematical literacy. Statistics is usually abandoned to “experts” because it is thought too difficult for most people to understand. Since this knowledge is also considered value-free, it is rarely questioned. In attempting to create an approach to mathematics education that can lead both to greater control over knowledge and to critical consciousness, it is important to have an adequate pedagogical theory that can guide and illuminate specific classroom practices. I want to argue that Paulo Freire’s “pedagogy of the oppressed” can provide the theoretical foundation for that practice.

Freire’s educational theory is complex. In this essay, I will focus on the problems he poses that are particularly pressing for teachers in schools in the United States. For this
reason, I will not treat his theory on why revolutionary party leaders must also be educators, or his assumptions (historically grounded in the reality of the various Third World countries in which he has practiced) that these leaders would come from the bourgeoisie, “committing suicide as a class in order to rise again as revolutionary workers” (Freire, 1978, p.16). Instead, I want to investigate his epistemology, his theory about the relationship between education and social change, and his methodology for developing critical consciousness. Because of Freire’s argument that critical education involves problem posing in which all involved are challenged to reconsider and recreate their prior knowledge, this presentation should be seen as an exploration intended to help extend our thinking, not as “Freire’s definitive formulas-for-liberation.” A discussion of my own experience teaching urban working-class adults\(^1\) basic mathematics and statistics for the social sciences demonstrates ways in which Freire’s theory can illuminate specific problems and solutions in critical teaching, and ways in which mathematics education can contribute to liberatory social change.

The rest of this article can be found here:

https://docs.google.com/viewer?a=v&q=cache:Q9xVO8G4uskJ:people.exeter.ac.uk/PErnest/pome25/Marilyn%2520Frankenstein%2520Critical%2520Mathematics.doc+&hl=en&gl=us&pid=bl&srcid=ADGEEShBhiKrEl1zklhEb_Ft3mXhu6fi oBLpI4LKuzKPYL6MSroro_VPeaSoSEGskc7Hf5h1voLa9eTumNiy0v9wmitXGHFp Wm0vgEnonBiwlwluJktdhOTsbmSid6dDUHt_Odi3We6U&sig=AHIEtbQdPvXrt9CnFkwck3V1roRntwQ3sw

\(^1\) The students at my school are adults who have a clear commitment to work in public or community service. Their average age is 35, about 70 percent are women, and about 30 percent are people of color.
APPENDIX E

Railside School Handout

Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School

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MEGAN STAPLES
University of Connecticut

Background/Context: School tracking practices have been documented repeatedly as having negative effects on students’ identity development and attainment, particularly for those students placed in lower tracks. Despite this documentation, tracking persists as a normative practice in American high schools, perhaps in part because we have few models of how departments and teachers can successfully organize instruction in heterogeneous, high school mathematics classes. This paper offers one such model through a qualitative and quantitative analysis.

Purposes of Study: In an effort to better the field’s understanding of equitable and successful teaching, we conducted a longitudinal study of three high schools. At one school, Railside, students demonstrated greater gains in achievement than students at the other two schools and higher overall achievement on a number of measures. Furthermore, achievement gaps among various ethnic groups at Railside that were present on incoming assessments disappeared in nearly all cases by the end of the second year. This paper provides an analysis of Railside’s success and identifies factors that contributed to this success.

Participants: Participants included approximately 700 students as they progressed through three California high schools. Railside was an urban high school with an ethnically, linguistically, and economically diverse student body. Greendale was situated in a coastal community with a more homogeneous, primarily White student body. Hilltop was a rural high school with primarily White and Latino/a students.

Research Design: This longitudinal, multiple case study employed mixed methods. Three
schools were chosen to offer a range of curricular programs and varied student populations. Student achievement and attitudinal data were evaluated using statistical techniques, whereas teacher and student practices were documented using qualitative analytic techniques such as coding.

**Findings/Results:** One of the findings of the study was the success of Railside school, where the mathematics department taught heterogeneous classes using a reform-oriented approach. Compared with the other two schools in the study, Railside students learned more, enjoyed mathematics more and progressed to higher mathematics levels. This paper presents large-scale evidence of these important achievements and provides detailed analyses of the ways that the Railside teachers brought them about, with a focus on the teaching and learning interactions within the classrooms.

The rest of this article can be found here:
I. Activity Principle

RME gives students the opportunity to actively participate in their mathematics learning process. According to Freudenthal (1973), “the students, instead of being receivers of ready-made mathematics, are treated as active participants in the educational process, in which they develop all sorts of mathematical tools and insights by themselves” (as cited by Heuvel-Panhuizen, 2000, p. 5). In other words, mathematics should be presented to the students in a manner where they can investigate and produce their own understanding of the content as this process ensures understanding of the subject rather than short-term memorization of concepts.

II. Reality Principle

In Dutch, the word zich realiseren means to imagine, and so the term 'realistic' refers to situations which can be imagined (Heuvel-Panhuizen, 2003). RME initially presents knowledge within such a concrete context allowing pupils to develop informal strategies, but gradually through the process of guided 'mathematization', allows students to progress to more formal, abstract, standard strategies. (Note that these contexts are chosen to help students' mathematical development, not simply because they are interesting!) As Bell and Shiu have suggested 'Abstract relationships are expressed by symbol-systems, and rules are developed for the manipulation...meaning can only be restored to the manipulations by recognizing the underlying concepts' (1981:1). It is where no meaning is offered that misconceptions arise. The RME problems, set in real world contexts, are presented, so that along with giving meaning and making mathematics more accessible to learners, they also illustrate the countless ways in which mathematics can be applied. (para. 18)

II. 'Mathematization' Principle

a. Horizontal mathematization: This is suggested by Harrison (2003) to be the students' discovery of mathematical tools that can help to organize and solve a problem located in a real-life situation.

b. Vertical Mathematization: This refers to the 'building up' to create more
challenging mathematics and hence to a greater use of abstract strategies.

III. Use of effective models Principle

The use of various models is key in RME as they “serve as an important device for bridging this gap between informal, context-related mathematics and more formal mathematics” (Heuvel-Panhuizen, 2000, p.5) that will eventually take the students to a level of vertical mathematization. In order to fulfill the bridging function between the informal and formal levels, models have to shift from a ‘model of’ a particular situation to a ‘model for’ all kinds of other, but equivalent, situations. An important requirement for having models functioning in this way is that they are rooted in concrete situations and that they are also flexible enough to be useful in higher levels of mathematical activities. This means that the models will provide the students with a foothold during the process of vertical mathematization, without obstructing the path back to the source. Examples of models are ratio tables and combination charts.

IV. Guidance Principle

As indicated by Gravemeijer et al. (Article 2), this implies beginning with the range of informal strategies provided by students, and building on these to promote the materialization of more sophisticated ways of symbolizing and understanding. Due to students’ directing the course of lessons, RME requires a highly ‘constructivist’ approach to teaching, in which children are no longer seen as receivers of knowledge but the makers of it’ (Nickson, 2000:5), and the role of the teacher is that of a facilitator. Allowing the students to begin at the basics, using informal strategies and constructing the mathematics for themselves, simulates the discovery of the mathematics and allows them to appreciate the complexity of the mathematics.

V. Interaction Principle

Education should offer students opportunities to share their strategies and inventions with each other. By listening to what others find out and discussing these findings, the students can get ideas for improving their strategies. Moreover, the interaction can evoke reflection, which enables the students to reach a higher level of understanding. The diagram below shows how knowledge and deep understanding are closely related when students are given the opportunity to interact and engage in math dialogue.
Background knowledge

New Idea

Shallow Understanding

Confusion

Talking and working together

Deep Understanding
APPENDIX G

Hans Freudenthal and Guided Tour of Netherlands Handouts

Hans Freudenthal: a mathematician on didactics and curriculum theory

K. GRAVEMEIJER and J. TERWEL

The main ideas in the work of Hans Freudenthal (1905–1990), the Dutch mathematician and mathematics educator, related to curriculum theory and didactics are described. Freudenthal’s educational credo, ‘mathematics as a human activity’, is explored. From this pedagogical point of departure, Freudenthal’s criticism of educational research and educational theories is sketched and fleshed out. Freudenthal’s approaches to mathematics education, developmental research and curriculum development can be seen as alternatives to the mainstream ‘Anglo-Saxon’ approaches to curriculum theory.

During his professional life, Hans Freudenthal’s views contradicted almost every contemporary approach to educational ‘reform’: the ‘new’ mathematics, operationalized objectives, rigid forms of assessment, standardized quantitative empirical research, a strict division of labour between curriculum research and development, or between development and implementation. Looking back from the present, it is of great interest to see how his ideas, which may at the time have seemed to embody recalcitrance for its own sake, have now become widely accepted. It would, of course, be far-fetched to suggest that this correlation implies a causal relationship, but it does indicate Hans Freudenthal’s special role, not only in mathematics education, but also in the development of curriculum theory and research methodology.

Hans Freudenthal had already earned his spurs as a research mathematician when he developed an interest in mathematics education and made himself acquainted with educational and psychological traditions in Europe and the US. Today, he is probably best known as one of the most influential mathematics educators of his time. In this paper, we shall try to highlight some of Freudenthal’s main ideas, while acknowledging that we cannot do justice to his wide-ranging work – even if we were able to. Our point of view will centre on curriculum theory and pedagogy, and we

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The remainder of this article can be found here:
www.tandfonline.com/doi/pdf/10.1080/00220270050167170
Mathematics education in the Netherlands: A guided tour

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1 Introduction

This paper addresses mathematics education in the Netherlands and provides a guided tour through the main aspects of the Dutch approach to mathematics education. The title of the paper also refers to the guidance aspects of mathematics education, the role of the teacher and of the curriculum. The tour will focus on the number strand in primary school mathematics. The two main questions to be dealt with are:

1. How is arithmetic taught in primary school in the Netherlands?
2. What contains the Dutch arithmetic curriculum?

Figure 1: The cover of the 'How the Dutch do arithmetic' report

About fifteen years ago, the first question was also investigated in a study called 'How the Dutch do arithmetic' (see Figure 1) (Van den Heuvel-Panhuizen and Goffree, 1986).

1 An earlier version of this paper was presented at the Research Conference on 'Teaching Arithmetic in England and the Netherlands' (Homerton College, University of Cambridge, 26-27 March 1999). A shortened version of this paper is published in Anghileri (2001), see Van den Heuvel-Panhuizen (2001).
REFERENCES


