PROFESSIONAL DEVELOPMENT HANDBOOK
FOR
CONCEPTUAL UNDERSTANDING IN SECONDARY MATHEMATICS:
3 DAYS OF TRAINING

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CONCEPTUAL UNDERSTANDING IN SECONDARY MATHEMATICS:
3 DAYS OF TRAINING

A Project

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Abstract

of

PROFESSIONAL DEVELOPMENT HANDBOOK
FOR
CONCEPTUAL UNDERSTANDING IN SECONDARY MATHEMATICS:
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Susan Pamela Kizner

Teaching conceptually in mathematics, which draws on the underlying structure and logic of concepts, has shown to be successful for students’ knowledge construction. Traditional mathematics instruction often focuses on procedures and rote memorization of skills, and tends to lack conceptual understanding. The following professional development handbook focuses on promoting the use of conceptual understanding in secondary mathematics, which in turn should improve students’ knowledge construction in mathematics. Drawing from constructivist theory, Vygotsky’s (1978) sociocultural theory of learning and his (1997) notions on concept development, and Donald Schön’s (1987) reflection-in-action, I created a professional development handbook for secondary mathematics instructors that emphasizes the importance of conceptual understanding in mathematics as opposed to traditional mathematics instruction.

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Mark Stoner, Ph.D.  Date
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Professional development training sessions are often a requirement for teachers in many school districts. Some research has found training courses for teachers to be effective because they provide teachers with the proper skills necessary to teach certain subjects (Farrell & Oliveira, 1993; Angrist & Lavy, 2001). Specifically, Angrist & Lavy (2001) found that students’ test scores increased when they had a teacher who went through an in-service training program compared to students whose teachers had not been through the same training. Since teacher training programs have the potential to influence student success, it is important to consider the kinds of topics that these programs should focus on, as well as how to properly construct training sessions that meet the needs of participants.

In mathematics classes, teaching conceptually, which draws on the understanding of the underlying structure and logic of concepts, has shown to be successful to students’ knowledge construction (Martinie & Bay-Williams, 2003; Ploger & Hecht, 2009; Vicich, Knott, & Evitts, 2007). Teaching conceptually in mathematics, as opposed to procedurally, involves drawing upon explanations for why certain procedures work, and helps facilitate the mathematical representation phase of word problems (Hiebert & Carpenter, 1992). Conceptual understanding in mathematics is important because it promotes critical thinking skills, allows students to retain and transfer information, and
reinforces the explanations behind procedures—as opposed to rote memorization of rules (Silver et al., 2009).

This paper accompanies the following professional development handbook titled *Conceptual Understanding in Secondary Mathematics: 3 Days of Training*. The training sessions outlined in the handbook are intended to be facilitated by skilled teachers, who have been trained in this approach, who buy into the idea of conceptual understanding in mathematics, and who are familiar with conceptual understanding in mathematics—presenters should not only be able to effectively manage the lectures, discussions, and collaborative activities involved, they should also create dialogue surrounding participants’ questions and concerns and promote collaboration among participants. The handbook focuses on promoting the use of conceptual understanding in secondary mathematics, which in turn should improve students’ knowledge construction in mathematics. Drawing from constructivist theory, Vygotsky’s (1978) sociocultural theory of learning and his (1997) notions on concept development, and Donald Schön’s (1987) notion of reflection-in-action, this paper reflects on the construction of the handbook itself, and also emphasizes the importance of conceptual understanding in the classroom and the need to move away from traditional mathematics teaching.

**Rationale**

A balanced math program has three interdependent components: Computational skills (“how” the math works), conceptual understanding (“why” the math works), and problem solving (“where” the math works) (Pearson Prentice Hall SB472 Training
Manual, 2009). Conceptual understanding, specifically, seems to be extremely important for knowledge construction—when students fail to understand the structure and logic of mathematics, they tend to use procedures incorrectly and are unable to detect when they have committed a procedural error (Martinie & Bay-Williams, 2003). Thus, the incorporation of conceptual understanding in the classroom is critical; the ideas presented in this handbook for professional development training seek to encourage participants in the 3-day training sessions to recognize and understand the importance of conceptual understanding and thus incorporate the techniques that accompany conceptual understanding into their teaching.

The handbook is designed to facilitate the expansion of math teachers’ pedagogical repertoire through 3 days of training sessions on conceptual understanding. Presenters should use the lectures, activities, and discussion topics as a guide for each day of training, and should most importantly encourage participants to use dialogue as a way to collaboratively reflect on their own teaching practices as well as create discussion surrounding new pedagogy that they plan to use in their classrooms. Over the three days of training, participants will gain a clear understanding of the importance of conceptual understanding in mathematics teaching, they will analyze math problems to see how ‘balanced’ they are, and they will create lessons that feature hands-on activities to promote conceptual understanding on given mathematics topics. Participants will leave the training with specific strategies and activities that they can realistically incorporate into their own teaching practices to increase students’ conceptual understanding—and thus, students’ knowledge construction—in mathematics.
Chapter 2
BACKGROUND OF THE PROJECT
Review of Literature

Traditional Mathematics Teaching

Most data on mathematics instruction in the United States comes from studies that involve direct observation of classroom teaching and studies that use teacher self-reported data from surveys (Silver et al., 2009). Across various studies of mathematics instruction, consistent overarching themes recur:

Mathematics instruction and instructional tasks tend to emphasize low-level rather than high-level cognitive processes (i.e. memorizing and recalling facts and procedures rather than reasoning about and connecting ideas or solving complex problems), require students to work alone and in silence (with little opportunity for discussion and collaboration), focus attention on a narrow band of mathematics content (i.e. arithmetic in the elementary and middle grades), and do little to help students develop a deep understanding of mathematical ideas (rarely asking for explanations using physical models, or calling for connections to real-world situations).

(Silver et al., 2009, p. 503).

Traditional mathematics teaching is often marked by a pattern that
Edwards and Westgate (1994) called the *essential teaching exchange*, wherein the teacher asks a question, a student provides a response, and the teacher offers evaluative feedback. When the teacher only offers evaluative feedback (and does not ‘probe’ for further explanation), students are left with a “right” or “wrong” answer with little to no explanation as to why it may be correct or incorrect. Many mathematics problems only have one correct answer, so it may seem difficult to invite discussion surrounding math concepts that only have one solution. However, though there may only be one solution, there are, most likely, multiple ways to solve the problem. Schoenfeld (1985) taught mathematics in a way that attempted to show students how to think mathematically about the world—in that way, he did not have students stop at the answer. Rather, his goal was to have students understand the mathematical nature of things, which required more understanding than simply deciding on the correct answer.

Though there is not a direct link between the generally low levels of student achievement in mathematics (based on national test scores, etc) and the kind of mathematics teaching that traditionally occurs (as outlined above), it is difficult to deny the plausibility of a causal relationship between how teachers are teaching mathematics and consequent student achievement. There has been an increasing public demand for improving the quality of mathematics teaching and learning in the United States, and a “widely shared concern for enhancing opportunities for students to learn mathematics with understanding” (Hiebert & Carpenter, 1992, p. 70). Thus, teachers must consider how to move away from traditional mathematics teaching that often relies heavily on low-level student cognition, and move towards teaching conceptually that draws upon
students’ high-levels of thinking. In the following handbook, I chose to focus on the importance of incorporating conceptual understanding techniques into mathematics courses because it is an integral component to teaching mathematics and is also lacking in many traditional mathematics classes. Thus, I designed the handbook and its 3 days of training sessions to encourage teachers to promote conceptual understanding in their own classes.

Conceptual Understanding in the Classroom

Conceptual understanding in mathematics focuses on the use of a variety of resources to draw upon students’ higher-order thinking skills in a way that allows them to understand the underlying logic and reasoning behind mathematical concepts (Silver et al., 2009; Carpenter, Fennema, & Franke, 1996). Conceptual understanding can be called by various names, including: authentic instruction, ambitious instruction, higher order instruction, problem-solving instruction, sense-making instruction, teaching for understanding, and cognitively guided instruction. Perhaps the long list of names for conceptual understanding contributes to its lack of use, or perhaps many teachers are actually using the techniques described in conceptual understanding, but are calling it something else. Either way, the term ‘conceptual understanding’ is broad and seems to be under a sort of identity crisis; this literature review will clearly define the term conceptual understanding, as does the following handbook.

Research evidence has pointed to the benefits of teaching conceptually in mathematics because it moves away from the narrow view of traditional mathematics
teaching, based primarily on recall and low-level cognition (e.g. Carpenter, Fennema, & Franke, 1996; Hiebert & Carpenter, 1992; Ploger & Hecht, 2009; Martinie & Bay-Williams, 2003; Vicich, Knott, & Evitts, 2007). Teaching mathematics conceptually includes drawing upon concepts that are cognitively demanding, promoting peer collaboration and increased discourse among students, and encouraging “engagement with mathematical reasoning and explanation, consideration of real-world applications, and use of technology or physical models” (Silver et al., 2009, p. 503).

Mayer (2002) argued that two of the most important educational goals are to promote retention (students remember what they have learned) and transfer (students can make sense of and be able to see what they have learned, and use it in the future); consequently, he examined how teaching and assessing can be broadened to go beyond simply remembering. He argued that when the goal of instruction is to promote transfer, objectives should include cognitive processes associated with understanding, applying, analyzing, evaluating, and creating (the top 5 levels of Bloom’s (1956) taxonomy). In other words, teachers should conduct activities and ask questions that involve higher-order cognitive processes so students can properly transfer the information that they have learned. When math teachers only require students to recognize and recall information (which, as noted above, is highly used in traditional mathematics teaching), students will most likely not be able retain or transfer the information. Without a “problem-solving attitude,” as Shuell (2001) called it, students will not be active learners and will, thus, have difficulties retaining information.

If the learning task is seen as an activity that requires
memorization of facts, the student is likely to engage in rehearsal strategies, and the teacher is likely to encourage such study behavior and seek memorized responses when questions and/or testing students.

(Shuell, 2001, p.107).

There also seems to be a connection between the knowledge gained from conceptual understanding and the tacit knowledge that reflection-in-action brings about (Schön, 1987). Tacit knowledge, “that which cannot be articulated,” may be characteristic of many things that teachers do, but it is the “obligation of the teacher to make the tacit [knowledge] explicit” to his students—that is, students will be more successful when they begin to have explicit answers to questions like, “How do I know what I know?” and “How do I know the reasons for what I do?” (Shulman, 1988, p. 32).

Teaching conceptually aims to bring about explicit answers to those kinds of questions—it draws on why mathematical concepts work and why procedures are used a certain way in some cases and used differently in others. Teaching conceptually (and reflectively) gives students reasons to learn and understand.

Conceptual understanding could be achieved in a variety of ways in the classroom. For example, teachers could incorporate hands-on activities to accompany mathematical concepts, students could work in small groups to increase collaboration and meaningful dialogue surrounding mathematical concepts, teachers could incorporate technology into their lessons to draw upon students’ interests, and students could be
required to answer questions that draw upon their critical thinking skills (i.e. math problems in which they must analyze, synthesize, or create information).

Students have shown to perform better on fraction and decimal problems after engaging in hands-on activities that involved multiple visual representations and computer programs (Martinie & Bay-Williams, 2003; Ploger & Hecht, 2009). Vicich, Knott, & Evitts (2007) paired board work with oral presentations as an instructional tool to create opportunities for students to develop “convincing mathematical arguments” and to communicate those ideas to their peers, and found that students benefited from the experience of presenting mathematics in a public forum setting (p. 420).

Similarly, Carpenter, Fennema, and Franke (1996) found that when teachers participated in a CGI (cognitively guided instruction) teacher development program and consequently changed their means of instruction, it was directly related to increases in their students’ achievement. The CGI program focused on evolving the teacher’s role from simply demonstrating procedures to helping students build on their mathematical thinking by engaging students in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking. Overall, they found that developing an understanding of children’s mathematical thinking can be a “productive basis for helping teachers to make the fundamental changes called for in the current reform recommendations” (p. 3).

This handbook is designed to change or enhance how teachers view teaching mathematics and consequently change or enhance the ways in which they present topics to their students—the training sessions designed here will be successful if teachers begin
(or continue) to incorporate techniques in their classes that promotes students’ conceptual understanding.

Teacher Training Programs

Teacher training programs (often called ‘professional development training’) are used to connect instructional theory to practice (Newman, 1999), yet there are often incongruities between espoused theories and theories-in-use (Argyris & Schön, 1974). For example, there are conflicting theories about teaching mathematics; teachers are often told that (theoretically) students will be most successful when they learn from direct experience or collaboration, however traditional mathematics teaching does not usually draw on direct experience (Silver et al., 2009). Similarly, the idea that “every child is to be treated as a unique individual” has been largely researched in educational contexts, yet teachers are given such large classes and such rigid time constraints that the “individuality of the child is subordinated to maintaining the system” (Argyris & Schön, 1974). Teacher training programs should be used as a way to connect espoused theories and theories-in-use, and should intertwine instructional theory and practice.

Communication is the essential tool that will connect instructional theory and practice, and without the deliberate collaboration and dialogue between presenters and participants, the professional development trainings will most likely not be successful. Thus, I incorporated a vast amount of whole-class discussions and small-group cooperative activities to promote dialogue between participants and with the presenter. Stoner (2007) warns against using a particular theory-in-use called “information
transfer,”—a linear process in which the instructor gives the student information (see Figure 1 below). He proposes a new model in which instructors and students work together to jointly create meaning surrounding a particular topic (see Figure 2 below).

Figure 1: Information Transfer

Source (instructor) ⇒ Message (course content) ⇒ Receiver (student)

Figure 2: Joint Construction of Knowledge

The model proposed for the joint construction of knowledge (Figure 2) is exactly the kind of communication that needs to occur in teacher training programs, as a way for presenters and participants to collaboratively create meaning surrounding the given agenda. Shuell (2001) argues that “learning is an active, goal-oriented, cumulative process in which the learner plays a critical role” (p. 104). The same should be true for teacher training programs—participants must play a critical role in their learning process in order to leave the training sessions having truly learned something. Shuell (2001) uses a “problem-solving” metaphor as a way to think about teaching and learning—it is a “goal directed activity that requires an active search for [answers]” in which the problem solver is mentally active and engaged in the process (p. 102). The following handbook presents a clear problem for participants—the lack of conceptual understanding in
mathematics classes. Participants are encouraged to use problem-solving techniques in order to actively make decisions about how to promote the use of conceptual understanding in their classes, which allows them to become active learners.

Schön (1987) argues that participants in training programs (and similarly, students in classrooms) cannot be taught what they need to now, but rather they can be coached. “He has to see on his own behalf and in his own way the relations between means and methods employed and results achieved. Nobody else can see for him, and he can’t see just by being ‘told,’ although the right kind of telling may guide his seeing and thus help him see what he needs to see” (p. 17). Overall, teacher trainings should “provide a body of knowledge and a range of skills that will meet immediate professional needs,” and it is helpful when presenters are practicing teachers (Department of Education and Sciences, 1987, p. 2).

Whitty, Barton, and Pollard (1987) argued that teacher education training courses should have four important features: first, training should emphasize a concern with “educational aims and consequences as well as having regard to means, skills, and efficiency”; second, training should combine practical skills and an open-minded attitude so that one can reflect on oneself and challenge one’s own assumptions as well as those of others; third, training should emphasize a process in which teachers monitor and evaluate their own practice; and finally, training should attempt to relate theory and practice by recognizing the nature of Donald Schön’s notion of reflection-in-action (p. 175).

There is a perpetuated distinction between educational theory and practice—a
teacher will acquire the appropriate knowledge of the subject matter subsequently to be delivered to his pupils and he will learn how to deliver it by practical classroom experience, but this can often lead to “pedagogical torpor characteristic of delivery teaching” (Stones, 1992, p. 199). Schön’s (1988) proposals for teacher training programs offer opportunities for teachers to communicate and challenge their insights with their peers. Considering what research says about effective teaching practices—for example, classrooms should not serve as ‘dumping grounds’ for information and students should not be ‘receptacles’ of empty knowledge—teacher training programs should follow the same effective practices. Within the training session, participants need to collaborate with their peers and reflect on their own teaching practices in order to facilitate new understanding.

Loughran (2002) similarly noted the gap between theory and practice, so in an attempt to help teacher training programs, he theorized that reflective practice was one way to integrate the two in a meaningful way. Specifically, he attempted to show how “experience in teacher education can be influential in the development of effective reflective practice…and how effective reflective practice might be important in the development of one’s professional knowledge” (p. 40). He found that the development of knowledge through experience, as a result of reflective practice, lead to a “recognition and articulation of professional knowledge” that intertwined theory and practice (p. 41). Thus, reflection on practice and experience is crucial in teacher training programs.

Stoner’s (2005) *Training for Change* manual provides “experienced trainers with some practical ideas for maximizing training outcomes,” including setting goals, asking
effective questions, and using time effectively (p. ii). He argues that training sessions should not be designed around “giving information that can better be read,” but rather the time should be used as a way to promote the application of skills learned by “using case studies, role-plays, discussions, [and] collaborative problem-solving” (p. 1.1). The following handbook was designed based on a dialogic framework for interaction, in which trainer and participants work collaboratively to reach a shared understanding and construct meaning (Linell, 2001), so that the information does not simply get “given”.

Overall, the claims set forth above for what constitutes an effective training session focus on the trainer acting as a “coach” for the trainees, participants in training reflecting on their own teaching practices, and activities that have participants apply newly learned skills. The following handbook, thus, focuses on those fundamentally important components in order for trainers and participants to have the most meaningful training experience. As a way to link theory and practice—and not perpetuate the distinction between them—I deliberately chose discussion questions and small-group activities that participants could then replicate in their own classrooms with their students. That way, they will leave the training not just with information about conceptual understanding, but with actual activities that they experienced first-hand that they are then encouraged to use with their students.

Theoretical Framework: Constructivism, Vygotsky, and Schön

The theoretical framework underlying the choices that I made in constructing this handbook draws from constructivist theory, Vygotsky’s (1978) sociocultural theory of learning and Vygotsky’s (1997) claims about concept development, and Schön’s (1987)
Constructivist Theory

Allen, Carmona, Calvin, and Rowe (2000) defined constructivist theory “not as a theory about teaching but more as a theory about knowledge and learning” (p. 1). Constructivism refers to the construction of knowledge based on one’s existing understanding. Different perspectives of constructivism emphasize either individual cognitive processes or social co-constructions of knowledge, but all stress the role of the learners as a “responsible, active agent in his/her knowledge acquisition process” (Loyens & Gijbels, 2008, p. 352). Dialectic constructivism or social constructivism, specifically, focuses on the social interaction among learners, peers, and the instructor to create knowledge (Windschitl, 2002). According to constructivist theory, students begin the learning process by assessing their internal understanding of a topic, then they apply their knowledge, and finally they engage in “debate or dialogue with other learners in order to hypothesize, analyze, and synthesize the information,” and thus, come to a new understanding (Gueldenzoph, 2003, p. 175). In this way, understanding is a “function of knowledge construction and transformation, not merely information acquisition and accumulation” (Loyens & Gijbels, 2008, p. 352).

Constructivist theory asserts that students find meaning by active engagement with others, which is why using small group activities in the classroom is beneficial (Loyens & Gijbels, 2008). Similarly, critical theorists argue that knowledge is socially constructed—that is, it is created through the dialogue and social interactions that take
place in the classroom (Sprague, 1992; Young, 1992; Freire, 1970/2006).

Communication is a necessary condition for real conceptual transformation—without communication in the classroom, there is a diminished opportunity for understanding to occur. Bakhtin’s (1981) and Linell’s (2001) writings on dialogism also have similar claims—they strongly emphasized the necessity of a joint construction of knowledge and shared intentions in interaction. Thus, students must be given opportunities in the classroom to interact with their peers and with the teacher in order to create meaningful dialogue, and thus construct knowledge.

Alexander (2008) describes dialogic teaching as collective, reciprocal, supportive, cumulative, and purposeful, in which active student participation in classroom talk is necessary for understanding. Dialogic teaching “harnesses the power of talk to stimulate and extend pupils’ thinking and advance their learning and understanding” (Alexander, 2008). Dialogic teaching draws from a broad repertoire of strategies, and is not a singular pedagogical technique—rather, it is an “approach to talk in teaching and learning…[and] is grounded in research on the relationship between language, learning, thinking and understanding…on what makes for truly effective teaching” (Alexander, 2008).

Thus, in the following handbook, I deliberately use activities (for example, group discussion questions and small group collaborative exercises) that allow participants to engage in dialogue and interact with each other as a way to jointly construct knowledge surrounding mathematics instruction.

Overall, a dialogic approach to teaching focuses on language and communication
for knowledge construction. Though knowledge construction implies communication, Vygotsky’s notions show how talk is the most important element to concept formation and development.

Vygotsky’s Sociocultural Theory of Learning (1978) and Concept Development (1997)

Vygotsky’s (1978) sociocultural theory of learning, put simply, claims that learning happens collaboratively. Moreover, “learning collaboratively with others, particularly in instructional settings, precedes and shapes development” (Lantolf and Thorne, 1995). Vygotsky theorized that human mental activity is always mediated by symbolic means; in other words, people use symbolic tools—like language, for example—to “organize and control such mental processes as voluntary attention, logical problem-solving, planning and evaluation, voluntary memory, and intentional learning” (Lantolf, 1994). Children begin to carry out symbolically mediated mental functions while under the guidance of others (for example, their parents, siblings, teachers, etc.), who initially assume most of the responsibility for carrying out the tasks. Over time, however, children assume increased responsibility for organizing their own mental activity and ultimately attain the ability to function independently of the other’s guidance (Lantolf, 1994). Most importantly, the ways in which children experience guidance as they carry out specific tasks is through dialogic interaction.

Vygotsky (1978) coined the term “zone of proximal development” (ZPD) to refer to the “distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem
solving under adult guidance or in collaboration with more capable peers” (p. 86). As noted above, the notion lies in the belief that children (or students, when referring to a classroom setting) initially need assistance or guidance from an expert (for example, the teacher or a more capable peer), but as they continue to work collaboratively with the expert they will gain the skills needed to work independently.

Burbules (1993) describes Vygotsky’s notion of “zone of proximal development” as a “state of readiness in which a student will be able to make certain kinds of conceptual connections. …Anything too simple for the student will quickly become boring; anything too difficult will quickly becoming demoralizing” (p. 122). According to Vygotsky’s claims, dialogic instruction should focus on peer collaboration (in which novice and expert students work together) and teacher-student interactions that connect past learning to future development. In order to narrow a students’ zone of proximal development—and thus, promote independent thinking—teachers and peers should ask “stepwise questions that are challenging, but not too discouraging [for students]” (Burbules, 1993, p. 122). Dialogue promotes the co-construction of meaning, and it is through dialogue with one’s teacher or peers that allows students to eventually move from novice to expert.

In Vygotsky’s (1997) *Thought and Language*, he claims that “real concepts are impossible without words, and thinking in concepts does not exist beyond verbal thinking” (p. 107). Moreover, the use of words is necessary for concept formation, which is why it is necessary for participants in these training sessions to use dialogue with one another (and with the presenter) to form their understanding; the “central moment in
concept formation…is a specific use of words as function ‘tools’” (Vygotsky, 1997, p. 107). However, a concept is not developed by simply ‘using words’ or talking, but rather the meaningful dialogue that arises from collaboration with one’s peers.

The tasks with which society confronts an adolescent as he enters the cultural, professional, and civic world of adults undoubtedly become an important factor in the emergence of conceptual thinking. If the milieu presents no such tasks to the adolescent, makes no new demands on him, and does not stimulate his intellect by providing a sequence of new goals, his thinking fails to reach the highest stages, or reaches them with great delay.


This idea that the outside world influences how one forms concepts is important to consider. Context is an important factor to concept development, which is why the activities presented in this handbook necessarily connect participants to each other and to their environment. Moreover, participants are encouraged to consider the impact that context has on their own students’ concept developments, and create lesson plans for their classes that connect students to one another and to their surroundings.

Though this handbook is somewhat monologic in nature—that is, it provides presenters with information and specific activities to ‘give’ participants through various means—the training sessions are designed in a way that promotes dialogism. Through small-group activities and whole-group discussions with the presenter, participants will
be able to diminish their zones of proximal development and create meaning surrounding conceptual understanding in mathematics. At the start of the training sessions, some participants may be novice teachers or unaware of the details of conceptual understanding in mathematics; by the end of the training sessions, participants will be somewhat experts on the topic who could then share their understanding and experiences with their students and other novice teachers. In accordance with the sociocultural theory of learning, the training sessions presented here will only be successful if participants collaborate with one another and with the presenter. I deliberately chose the order of activities presented in the handbook: The start of the training draws more heavily on participants’ lower-levels of cognition in which the presenter works closely with the teachers, and the training ends with activities that draw upon participants’ high-levels of cognition in which they work collaboratively to create and evaluate their own projects. Vygotsky’s (1978) sociocultural theory of learning and his (1997) notions on concept development allowed me to organize the handbook in a way that moved up through the levels of Bloom’s taxonomy, and it also made me closely evaluate every activity to make sure that collaboration was present and dialogue was encouraged.

Schön’s Reflection-in-Action

Donald Schön (1987) argues that reflection-in-action in teaching and learning is important because it “rethinks some part of our knowing-in-action [that] leads to on-the-spot experiment and further thinking that affects what we do” (p. 29). He uses ‘knowing-in-action’ to refer to a “smooth sequence of activity, recognition, decision, and
adjustment” that happens without much thought; however, when that familiar routine produces an unexpected result, or an element of surprise, one may respond to it by reflection (p. 26). Schön’s (1987) notion of reflection-in-action underlines some of the decisions I made in the construction of this handbook, and also underlines the importance of conceptual understanding in mathematics instruction.

Reflection-in-action is a response to a surprise in routine responses, and questions the structure of one’s current knowledge. Reflecting-in-action consequently reshapes what one is doing while he or she is doing it, and “gives rise to on-the-spot experiment” (Schön, 1987, p. 28). In an instructional context, it is important for teachers to use reflection-in-action in order to respond to students in a way that helps them “get over their particular difficulties in understanding something, build on what they already know, and discover what they already know but cannot say” (Schön, 1988, p. 19). Schön urges teachers to use reflective teaching in their practices because it gives students reasons to ask why they know what they do. A similar idea of encouraging students to ask “why” things work in mathematics (and encouraging teachers to ask those kinds of questions of their students) is the main idea presented in the following handbook on conceptual understanding.

One of the ways that teachers can build an inclination toward reflecting teaching is through ‘instructional supervision’ and “any activity that supports, guides, or encourages teachers in their reflective teaching,” including in-service teacher training programs, like the one implemented here (Schön, 1988, p. 21). In a teacher training program, the trainers or presenters become ‘reflective supervisors’ or ‘coaches’ for the
teachers involved; coaching reflective teaching (or conceptual understanding, in this case) involves three things: 1) making sense of and responding to the “substantive issue of learning/teaching in the situation at hand,” 2) entering into the teacher’s ways of thinking about it, and 3) doing the aforementioned tasks in a way that makes defensiveness less likely (Schön, 1988, p. 23).

Trainers can accomplish the three tasks above in various ways; for example, it may be helpful for trainers to share personal experiences from their classes as well as having teachers share their own experiences. In that way, the trainer joins the teacher in reflecting on the teacher’s own reflection-in-action, and they enter a “kind of collaborative on-the-spot research” (Schön, 1988, p. 23). Overall, reflective teaching and conceptual teaching in mathematics are both important for student success—thus, teachers must be given opportunities to learn how to become reflective teachers and to incorporate conceptual teaching techniques into their current practices. The following handbook contains activities that have participants reflect on their own teaching practices, as well activities that have participants work together towards a shared understanding of how to teach conceptually in mathematics.

This Handbook

This handbook is intended for trainers to use as a guide to assist them in running the professional development training sessions on conceptual understanding in mathematics. There may be parts of the handbook that do not directly fit the needs of the participants involved, and should thus be shaped to fit the needs of individual teachers. Any of the lectures, discussions, and activities could easily be modified and
individualized to meet the requirements of teachers at a particular school site or in a particular school district. The training sessions are geared toward secondary mathematics instructors (middle school and high school math teachers), but the ideas presented could be useful to elementary school mathematics teachers as well—some slight modifications to the activities and discussion questions will be needed.

Most importantly, presenters need to focus on the necessary communication and dialogue involved in the training sessions in order for the discussions and activities to be meaningful, and in order for participants to jointly create an understanding surrounding the need to teach conceptually in their classes. Drawing from constructivist theory, Vygotsky’s (1978) sociocultural theory of learning and his (1997) notions on concept development, and Schön’s (1987) notion of reflection-in-action, the activities in the handbook are deliberately designed in a way to promote collaboration, talk, and reflection among participants. It is the presenter’s responsibility to modify any of the suggested activities in the handbook to allow for meaningful collaboration and reflection.

Agenda

Having an agenda that covers the key concepts for the day is important, and if the agenda is managed properly it will be beneficial for the trainer as well as the participants (Tropman, 2003). Presenting the agenda to the participants is critical because audiences “need to have a feel for the ‘shape’ of the presentation” (Garmston & Wellman, 1992). Specifically, making copies of the agenda for participants will be helpful—that way participants can see the kinds of activities that are scheduled for the day, approximately
how long they should spend on each activity, and they will know what to expect from the overall session. The timeline on each agenda (for Days 1, 2, and 3) should serve as a rough estimate for how long each activity should take. Following the agenda closely will ultimately reduce participant anxiety and should aid group members in achieving high quality group decisions (Tropman, 2003).

Specify Goals

Shuell (2001) suggests that in order for students to successfully learn the desired material, they should have “some idea of the desired goal of the learning activities in which they are engaged” (p. 104). Clearly stating the goals at the beginning of each session is important and should ultimately help the flow of the session. Before going into the training sessions, trainers need to consider what they want participants to leave with.

Garmston & Wellman (1992) encourage presenters to look at their intended outcomes from two perspectives: the macro (bird’s eye) view and the micro (worm’s eye) view. From the macro perspective, presenters consider outcomes that resonate from “core values, concerns, passions, and missions in life” (p. 4), while micro-outcomes relate specifically to the presentation topic. When presenters consider intended outcomes from both perspectives, they are able to make conscious decisions during the presentation to shape the needs of the participants. Each day of training has an accompanying agenda with objectives for the day, which are written from a micro-perspective; that is, they relate specifically to the presentation topic and measurable outcomes regarding the specific mathematics techniques that the training focuses on. I also included a list of
“macro objectives for professional development on conceptual understanding,” which presenters are encouraged to share with participants at the beginning of the first day of training, and then refer back to at the conclusion of the 3rd day of training. These objectives focus on how the training sessions affect participants’ outcomes for days, months, or years after the trainings are complete. By revisiting the list of “macro objectives” at the conclusion of the training sessions, participants should recognize that they not only met the micro-objectives set forth everyday (for example, creating hands-on activities), but that those micro-objectives lead to something greater—that they will continue to use the techniques they learned here and continue to build off of those techniques in order to create a new culture in their classes that promotes conceptual understanding.

Clearly stating the goals and expectations for the day will prepare the participants to listen and engage in activities. Giving participants a preview of the session “states the aim of the [presentation] (specific purpose) and how you plan to accomplish it (pattern or structure of ideas)… It creates listening categories, so students know what to listen for” (Kougl, 1997, p. 304). Thus, presenters are encouraged to photocopy and pass out the written agenda and objectives for each day of training, and review the day’s objectives with participants before starting each session.

Lecture

A lecture on defining conceptual understanding and where it fits in with a balanced math program is incorporated into the handbook. A lecture is appropriate for
getting that particular information across because a) the objective for that part of the training is to give the participants information, b) the information must be organized in a particular way (i.e. the Venn diagram), c) it is necessary to arouse interest in the subject, and d) it is necessary to introduce the topic before participants continue to engage in activities surrounding the information (Kougl, 1997). The lecture should, hopefully, aid thinking—participants may not have heard the term ‘conceptual understanding’ before this training session, and the lecture is intended to give them information about it so they can begin thinking about it on their own.

Since the lecture serves as a jumping-off point for the rest of the training session, it is important that participants listen so they can, in turn, start to formulate new ideas on their own. Participants are most likely motivated by “their own interests, active involvement, novelty, choice making, peer interaction, game-like features, and a sense of accomplishment,” so this lecture is aimed at making the information personally relevant to participants, it uses new material that is “mentally stimulating” and has “emotional impact,” it is organized clearly to aid student listening, and it allows opportunities for students’ active responses (Kougl, 1997, p. 287).

Discussion

There are various discussion sections throughout the handbook. One of the discussions on Day 1, for example, revolves around current mathematics teaching practices and includes a list of ‘discussion questions’ and possible responses. Presenters may add more questions to the discussion to best fit the needs of participants in their
training session, or perhaps participants will bring up some questions of their own. To get the most out of the discussion, presenters must be open to discussion, encourage creative ideas, and create a positive and supportive “talk friendly environment” (Kougl, 1997, p. 210). Similarly, the physical environment should invite discussion—the presenter should think of the different options available for the best spatial arrangement that will allow participants to look at each other while they communicate.

The posed discussion questions in this handbook are sequenced in a way that move up the hierarchy of Bloom’s taxonomy (1956) in an inductive sequence—for example, the first question (from the first whole-class discussion on Day 1 surrounding traditional mathematics instruction) asks participants to recall from personal experience how mathematics is traditionally taught (knowledge), the second questions asks participants to analyze themselves or other teachers in their math department (analysis), and the last question asks participants to determine why moving away from traditional mathematics teaching may be beneficial to student success (evaluation). The inductive sequencing makes sense here because participants need to recognize what traditional mathematics teaching is (i.e. recall) before they evaluate why it may not be helpful to students’ knowledge construction.

The questions used throughout the handbook are designed primarily as open questions—there are no ‘right’ or ‘wrong’ answers, and any response is appropriate within the general boundaries (Kougl, 1997). Those kinds of questions “invite longer, elaborated, personalized responses…[and] encourage more complex thinking,” so presenters should use each discussion question as an opportunity to let participants
respond as much as possible (p. 219). It may be helpful for presenters to use secondary questions to get even more responses out of participants.

Silvernail (1979) found that teachers who asked questions frequently, asked a mix of lower-order and higher-order questions, used secondary probing questions, and reaffirmed students answers, were more effective than those who did not. Similarly, Wilen (1982) outlined questioning techniques for teachers to use to best shape discussion in the classroom. For example, when creating discussion questions, teachers should: 1) plan key questions to provide structure to the lesson, 2) phrase questions clearly and specifically, 3) adapt questions to student ability level, 4) ask questions logically and sequentially, 5) ask questions at a variety of levels and 6) use questions that encourage wide student participation. The discussion questions outlined in this handbook follow Wilen’s (1982) questioning techniques fairly well, so hopefully they will prove to be effective during discussion. As part of his ‘questioning techniques,’ Wilen (1982) also included that teachers should follow up student responses, give them time to think when responding, and encourage student questions. The specific questions provided should hopefully guide discussion well, but it is ultimately up to the trainer to manage the discussion and students’ responses effectively.

Overall, the various discussion sections in this handbook are designed for presenters to gain information from their specific participants, and to use that information to shape the rest of the training sessions. For example, the presenter may find out during the first discussion that all math teachers at the training already use conceptual understanding techniques—in that case, the presenter will have to make some alterations
to some of the activities in the handbook to best fit their needs. On the other hand, the presenter may find out during the first discussion that none of the teachers at the training use conceptual understanding techniques—in that case, the presenter may need to consider which activities should be given more time. If discussion is conducted properly (i.e. the room is physically inviting for discussion, the climate is positive and supportive, the questions are asked in the given inductive sequence, and the presenter effectively manages participant responses), it will serve as a basis for knowledge construction for the trainer and participants.

Collaborative Learning Activities

In a cooperative or collaborative structure, students work together toward a shared goal, and personal success depends on the other group members (Kougl, 1997). This handbook outlines various small group collaborative activities that should hopefully allow participants to draw on each others’ ideas and expand upon their own knowledge.

In one small group activity, participants will analyze a math worksheet to see how ‘balanced’ it is, referring to the Venn diagram of a balanced math program. Participants will work collaboratively to unanimously make decisions about where each math problem falls within the Venn diagram. A collaborative learning environment is appropriate for this activity because it involves careful analysis and draws on participants’ critical thinking skills—presenters should remain fairly absent during this activity (except for circulating the room and answering clarifying questions about the activity) so participants can engage in meaningful dialogue with each other surrounding
the mathematical concepts on the given worksheet. There are no ‘right’ or ‘wrong’ answers to this activity, and some of the questions may be slightly ambiguous; in that way, it will be beneficial for participants to work together to discuss why they made certain decisions regarding the math worksheet. Rather than having the trainer tell participants how to correctly analyze each problem (which would be completely one-sided), and rather than conducting a whole-class discussion surrounding the worksheet (which may bring about interesting opinions, but would most likely not include all participants), the collaborative activity allows all participants to contribute their ideas easily with one another and each group will ultimately share their unanimous decisions with the rest of the class.

In the following collaborative activities, participants create an introductory lesson and a follow-up lesson that incorporates hands-on activities as a way to promote conceptual understanding. As with the previous activity, participants are encouraged to work together toward a shared goal and to draw upon each other’s previous knowledge and critical thinking skills. The task of creating a lesson plan could have been assigned as an individual assignment, but the “synergistic effect of working together produces greater outcomes than an individual working alone”—that is, the new creative ideas that arise out of working collaboratively and building off of each others’ ideas will theoretically be of higher quality than a singular individualistic idea (Kougl, 1997, p. 236).

Groups should remain fairly small (no more than 4 participants to a group) so all group members have opportunities to contribute to the conversation and the task at hand.
For the second activity, in which participants create a lesson plan, it will be beneficial for participants to be grouped by the subject area that they teach—for example, a group of all algebra teachers or a group of all geometry teachers. That way, participants can relate to each other’s specific experiences and collaborate on course-specific math concepts.
Chapter 3

CONCLUSION

This paper outlines the featured components in the following handbook titled *Conceptual Understanding in Secondary Mathematics: 3 Days of Training*, as well as the theoretical framework that was used to make decisions in the handbook, and also focuses on the importance of promoting conceptual understanding in mathematics.

Future Research

Future studies, and subsequently future professional development trainings, should focus on specific techniques or practices that could be used in conjunction with teaching conceptually to enhance student success. One might consider how to best fit the needs of English language learners in mathematics classes, which will most likely involve more specific techniques in promoting their conceptual understanding—for example, it may be helpful to group students together who speak the same primary language for a collaborative activity. Similarly, it would be interesting to examine the specific techniques that should be used to promote conceptual understanding for low-achievers, middle-achievers, and high-achievers in mathematics. Teaching conceptually should increase students’ knowledge construction in mathematics, so we must consider how to teach conceptually in a way that fits *all* of our students’ individual needs.

Though this handbook focuses on secondary mathematics instruction (including pre-algebra, algebra, geometry, algebra II, trigonometry, pre-calculus, calculus, and various other middle school and high school mathematics courses), these trainings can
also be modified to fit elementary mathematics instruction (for example, 2nd through 6th grade mathematics). Students are first introduced to integers, basic operations (including adding, subtracting, multiplying, and dividing), decimals, and fractions in elementary school, and a conceptual approach to those topics would certainly yield the same benefits as conceptual teaching in secondary mathematics. Perhaps teaching conceptually in elementary school is even more useful because it lays the groundwork for how students will view and understand mathematics in future courses.

In addition to mathematics instruction, it would be useful to consider the potential affects of conceptual understanding in other subject areas, for example science and social studies. Any classroom setting in which instructors are drawing on students’ critical thinking skills and not solely relying on students’ rote memorization, should be beneficial to student knowledge construction. It would be interesting and useful to find out what specific techniques are involved in teaching science, social science, or other subjects conceptually, and how teachers could realistically begin to incorporate those techniques into their current pedagogy.

**Summary**

This paper accompanies the following professional development handbook titled *Conceptual Understanding in Secondary Mathematics: 3 Days of Training*. For the training sessions to be successful and for the lecture, discussions sections, and collaborative activities to be run effectively, the professional development should be conducted by a skilled presenter and teacher, who is already familiar with conceptual understanding and who buys into the idea that conceptual understanding is critical for
student success. Drawing from constructivist theory, Vygotsky’s (1978) sociocultural theory of learning and his (1997) notions on concept development, and Donald Schön’s (1987) notion of reflection-in-action, this paper *not only* focuses on the need for joint knowledge construction and reflective teaching through the use of dialogue and collaborative activities in training sessions, but *also* emphasizes the importance of conceptual understanding in the mathematics classroom and the need to move away from traditional mathematics teaching. Specifically, teachers need to go beyond using rote memorization of rules and procedures to teach mathematics concepts, and need to create opportunities for students to explain *why* math concepts work by drawing upon their critical thinking skills. Conceptual understanding could be achieved through the use of hands-on activities, connecting concepts to real-life applications and to students’ background interests, small group interactions in which meaningful dialogue is created, and by incorporating higher-order questions into everyday lessons.

Trainers who facilitate the professional development days should serve as coaches for the participants—they should reflect on their own teaching experiences and share them with participants, and have participants reflect on *their own* teaching experiences as well. Through reflection and collaboration, trainers and participants will work together to create meaningful dialogue and new knowledge surrounding the importance of conceptual understanding in mathematics, and novice teachers at the beginning of the training sessions can—with the help of the presenter—become experts in how to teach conceptually.
The professional development training sessions will be successful if participants leave having reflected on their own teaching practices, understanding the importance of conceptual understanding in secondary mathematics, and intending to use techniques in their classes to promote conceptual understanding.
Chapter 4

PROFESSIONAL DEVELOPMENT HANDBOOK

Professional Development Handbook

for

Conceptual Understanding in Secondary Mathematics:

3 Days of Training

Susan Pamela Kizner

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About the Author

I have been teaching mathematics at the secondary level in Sacramento since 2006, and I have not only grown to love teaching mathematics, but I have also acquired a special interest in mathematics pedagogy and the various ways in which teachers can enhance student understanding. I completed a master's degree at California State University, Sacramento in Communication Studies (with an emphasis on instructional communication) in December 2010, and I used my time in the master’s program studying and researching secondary mathematics instruction.

Through experience and research I have discovered that there seems to be a disparaging gap between espoused theories of what "good mathematics teaching" looks like and what is actually happening in math classes. Not only did I use the findings from my research projects to enhance my own teaching pedagogy, I also began thinking about how mathematics departments and instructors could improve student learning in mathematics. During the process of conducting and collecting research from my own classroom and other math teachers' classes, I bought into the idea of "conceptual understanding" in mathematics. I found that students are more likely to fully understand and consequently remember mathematics concepts when taught "conceptually," for example through the use of small group activities that draw upon their critical thinking skills and that focuses on the underlying logic behind mathematical processes.

I believe so deeply in teaching mathematics conceptually, that I completed this Professional Development Handbook focused solely on conceptual understanding in secondary mathematics, with the hope of teachers finding value in teaching conceptually, as I did through experience and research. There are many ways that conceptual understanding can be achieved in mathematics, and this handbook offers some suggestions and recommendations for instructors.

For questions, comments, or concerns regarding this handbook, please contact me at suzie-kizner@scusd.edu.
Introduction

This handbook is designed as a guideline for 3 6-hour days of professional development training on conceptual understanding in mathematics, and should be used to promote conceptual understanding in the classroom, which in turn should improve student success in mathematics. The trainings should ideally be comprised of secondary mathematics teachers (though some of the activities could be altered to fit elementary school mathematics, as well), and should be given over 3 consecutive days if possible. Group size should be managed so participants have the opportunity to fully engage in classroom discussion; a maximum of 30 teachers is ideal—for a bigger group (more than 30), it may be beneficial to break participants up into smaller groups for discussions.

This training should hopefully be a meaningful and useful experience for teachers, that will allow them to leave with completed lesson plans and hands-on activities that promote conceptual understanding on given mathematics topics. If teachers start using the techniques and strategies that are recommended from this training in their classrooms, they should see students improve in their classes. At the beginning of the first day of training, there is a section on the agenda titled “Why We are Here”; do not skip this section—participants need to understand why this training is important and how it can be useful to them in shaping the way they teach mathematics to their students. Any slight change in mathematics teaching that moves towards conceptual understanding will be beneficial to student success.

A balanced math program (according to the publishers of Prentice Hall
Mathematics textbooks) has three interdependent components: Computational skills (“how” the math works), conceptual understanding (“why” the math works), and problem solving (“where” the math works). Conceptual understanding specifically seems to be extremely important for knowledge construction—when students fail to understand the structure and logic of mathematics, they tend to use procedures incorrectly and are unable to detect when they have committed a procedural error. Thus, the incorporation of conceptual understanding in the classroom is not only recommended, it is critical.

The handbook is designed to facilitate the expansion of math teachers’ pedagogical repertoire through 3 days of training sessions on conceptual understanding. Presenters should use the lectures, activities, and discussion topics as a guide for each day of training, and should most importantly encourage participants to use dialogue as a way to collaboratively reflect on their own teaching practices as well as create discussion surrounding new pedagogy that they plan to use in their classes. I have included a complete lecture (during the first day) on defining conceptual understanding; it would be best for the presenter to read through the lecture first, possibly make some bullet points to guide him through the main ideas, and then give the lecture in his own words so it does not sound rehearsed, memorized, or simply read. Communication with and among participants is necessary for the training sessions (and its subsequent activities) to be successful and meaningful to participants. Most importantly, encourage participants to collaborate with one another in order to create a shared understanding regarding the importance of conceptual teaching in mathematics and how it can be achieved with their particular students.
The Federal No Child Left Behind Act (NCLB) puts pressure on American public schools to demonstrate significant increases in student achievement; specifically, large urban school districts (who primarily serve low-income and minority students) are under particularly great pressure to identify and implement educational programs that will address the needs of their students. This handbook should be used as a tool to help implement the use of conceptual teaching in secondary mathematics, as a way to aid schools in improving mathematics instruction and learning. Depending on your school site’s demographic information, funding, or possible program improvement status, you may choose to modify some of the discussions and activities presented here to best fit the needs of your students. For example, if your school has a large population of English language learners, it will most likely be beneficial for participants to create lessons that focus on language or vocabulary in mathematics.

Before jumping in to the first day of training, please post or pass out the “Macro Objectives for Professional Development on Conceptual Understanding” (page 6). This outlines the objectives for the entire training and views the training from a macro perspective—that is, the “bird’s eye” view of how this training will affect participants’ outcomes for days, months, or years after the trainings are complete. Having participants identify the macro objectives at the beginning of the training session will allow them to see that the techniques and strategies that they will learn here can affect how they will subsequently teach mathematics. Also pass out the table on Bloom’s Taxonomy (page 55) to accompany the macro objectives, since they refer to some of the levels of thinking in Bloom’s Taxonomy.
Each day of training also has an accompanying agenda with objectives for the day; please post or pass out the agenda and objectives at the beginning of each day of training, so participants know what to expect from the session. The agenda and objectives written for each day of training are from a micro-perspective; that is, they relate specifically to the presentation topic and measurable outcomes regarding the specific mathematics techniques that the training focuses on. During the last day of training, participants will refer back to the “macro objectives” and will predict how they will begin to incorporate the techniques they acquired through these trainings in their own classes. At this point, participants should recognize that they not only met the micro-objectives set forth everyday (for example, creating hands-on activities), but that those micro-objectives lead to something greater—that they will continue to use the techniques they learned here and continue to build off of those techniques in order to create a new culture in their classes that promotes conceptual understanding.

By the end of the trainings, teachers will have a clear understanding of the importance of conceptual understanding in mathematics teaching, and will leave with specific strategies and detailed lesson plans including hands-on activities (that hopefully they begin to utilize) on how to realistically achieve conceptual understanding.

In this handbook, the writing in *italics* is meant for the presenter—it offers explanations for and introductions to the various activities that will take place throughout the day. Any writing that is *not* in italics is meant as guidelines for what the presenter should do and say throughout the training sessions. To save paper, you may choose to rewrite some of the handouts on a whiteboard, blackboard, or large poster paper.
Macro Objectives for Professional Development on Conceptual Understanding

Though each day of training has accompanying objectives, they were created from a micro-perspective; that is, they focus on your immediate outcomes from the specific activities and discussions that you encounter during each day of training. Listed below are “macro objectives” for this professional development training—that is, these are the outcomes that may not occur immediately or may not even be measurable, but may affect the overall culture of your classroom and how you teach mathematics in the future.

Macro objectives for participants:

1) To buy into the idea that conceptual understanding is critical for student success in mathematics

2) To analyze your current teaching pedagogy, and subsequently expand your pedagogical repertoire

3) To think about the worksheets, quizzes, tests, lectures, and activities you give your students and decide if they have a “conceptual understanding component”.

4) To change the culture in your classroom from “memorization procedures” to a “collaborative exploration of math concepts.”

5) To move away from solely asking “remember” questions, to having students analyze, evaluate, and create (see Bloom’s Taxonomy handout).

6) To show other teachers at your school site (in the math department, and also the other departments) the value of conceptual understanding.

7) To identify the impact that teaching conceptually has had on you, your students, and the overall structure of your classes.
Day 1: Agenda

I. Introductions……………………………………………… 8:30 – 9:00
II. Why we’re here: The importance of conceptual understanding... 9:00 – 9:35
III. Goals for today……………………………………………… 9:35 – 9:40
IV. Current mathematics teaching……………………………………... 9:40 – 10:30
V. Defining conceptual understanding……………………………. 10:30 – 11:30
VI. Discussion: Promoting conceptual understanding……………… 11:30 – 12:00

LUNCH

VII. Activity: Analyzing a math worksheet……………………… 1:00 – 2:30
VIII. Summary of today’s session…………………………………….. 2:30 – 2:45
IX. Closure/Homework……………………………………………… 2:45 – 3:00

Main objectives for participants:

1) To understand the importance of conceptual understanding
2) To identify pedagogy representative of traditional mathematics instruction
3) To recall the definition of conceptual understanding and how it fits in to a ‘balanced math program’
4) To collaboratively create a list of possible techniques that can be used in the classroom as a way to promote conceptual understanding
5) To apply what we know about conceptual understanding and a balanced math program to analyze a math worksheet
I. Introductions (≈ 30 mins)

Start the day with basic introductions from participants. If the group is small enough, it may work best for participants to share their introductions with the entire group. For larger groups, participants can introduce themselves in small groups (4-5 people). Use the following questions as guidelines for how teachers can introduce themselves to the group:

a. Where do you teach?

b. What mathematics subject(s) do you teach?

c. How long have you been teaching?

d. What do you hope to get out of this professional development training?

i. Record participants’ responses on a large paper or whiteboard for everyone to see. You will soon refer back to this.

II. Why we’re here: The importance of conceptual understanding (≈ 35 mins)

Participants will most likely appreciate hearing why this professional development training is useful and why it is important. Regardless of whether the training is mandatory or voluntary, it will be helpful to relay the following information to participants at the beginning of the session:

a. This training

This training is designed to increase your understanding of conceptual teaching in mathematics and includes activities in which you will explore various techniques regarding conceptual understanding. The next two days will be spent reviewing the
importance of conceptual understanding and how it can realistically be achieved in your classroom. You will leave this training with a variety of lesson plans (that you and your peers will create), techniques, and strategies that promote conceptual understanding in mathematics.

b. Conceptual understanding

Conceptual understanding in mathematics is extremely important for student success; when students fail to understand the structure and logic of mathematics, they tend to use procedures incorrectly and are unable to detect when they have committed a procedural error. If we want our students to remember procedures from one day to the next (or from one year to the next), to be more comfortable solving (or at least attempting to solve) word problems, and to notice when they have committed procedural errors, then we need to incorporate conceptual teaching strategies into our classrooms.

c. Review participants’ expectations from introductions

At this point, refer back to the list of participants’ expectations for this training session. You should go through each item on the list (of what they ‘hoped’ to get out of this training) and decide if the next 2 days of training will (or will not) meet their expectations. For example, if a participant stated, “I hope to learn more strategies on teaching subtracting integers,” you can tell them that their expectation will most likely be met in the next 2 days. On the other hand, if a participant stated, “I hope to learn how to relate these ideas to the textbook,” you may want to tell them that though that is a realistic expectation from a mathematics training, this training specifically does not focus on using the textbook.
If a majority of participants seem to have the same (or similar) hopes from this training, consider making them a priority. For example, if many teachers hope to leave with a better understanding of how to use technology in the classroom to enhance student success, make sure that you spend considerable time on that topic.

At this point, you should invite discussion and questions from participants; perhaps some participants want to add more of their hopes/expectations to the list. You should encourage them to add as many ideas to the list as possible before continuing with the session.

III. Specify goals for the day (≈ 5 mins)

Clearly state today’s objectives to participants. State the goals in a way that tells participants what they will accomplish by the end of today’s session. For example:

Today you will:

a. Discuss, as a class, how most math classes are traditionally taught
b. Recall the definition of conceptual teaching and understanding
c. Create a list of possible strategies to promote conceptual understanding in mathematics
d. Analyze a math worksheet in a small group

IV. Current mathematics teaching (class discussion, ≈ 50 mins)
Rather than simply stating, “conceptual understanding is key to student success,” it is important to recognize what we are currently doing and why it may not be considered effective mathematics teaching. Follow the questions below to help guide the whole-class discussion, and consider the possible responses. Make the room physically inviting for discussion—create a large circle, or for a bigger group simply make sure that everyone can see everyone else. Be open to atypical responses and allow all participants to share their ideas with the group.

Based on how your current mathematics program runs (i.e. department funding, set leadership roles, group cohesiveness), possible responses will vary. A recorder for the group should write participants’ responses on a poster board (large enough for everyone to see) so you can connect similar ideas to one another, and so you can refer back to it later.

Discussion Questions:

- From your experience, if you had to describe traditional mathematics teaching, how would you describe it? (Consider the ways in which you teach, the ways in which your colleagues teach, and what you’ve heard from other mathematics teachers throughout the city, state, country, etc.)

  • Possible responses:
    - Lecture, homework, test, repeat
    - Memorize and study
    - Plug and chug
The teacher provides model examples to the students, then students work individually.

Group work

Worksheets

Considering the descriptions you just gave of ‘traditional mathematics teaching,’ do most mathematics teachers in your department use those techniques?

- Possible responses:
  - Yes. Many teachers lecture everyday, assign homework, and then give the test.
  - Some of them use those techniques. But lecturing is often necessary in mathematics courses.
  - No. As a department, we have agreed on using various projects and hands-on activities to teach many mathematical concepts.

Why, do you think, that moving away from traditional mathematics teaching may be helpful for student achievement in math?

- Possible responses:
  - It will be more exciting for them not to sit through lectures everyday, memorize formulas, and do mindless worksheets.
  - Students may be more likely to remember concepts if they engage in class activities that involve those concepts.

This final question should lead nicely into the following lecture on the definition of conceptual understanding.
V. Lecture: Defining conceptual understanding (≈ 1 hour)

Become familiar with the following lecture before the training session. It will be beneficial to read through the lecture and choose which topics best fit the needs of participants at your site. You should then create a lecture in your own words (based off of the one offered below), so it does not sound ‘read’ and insincere.

Start by showing the following Venn diagrams (on the next page) of how mathematics is traditionally taught (Diagram 1) versus how we hope to teach mathematics in the future (Diagram 2). It is important for participants to recognize the difference between how most mathematics classes are typically taught, and where they should be headed in the near future. The diagram should explicitly show a shift from an overuse of ‘procedural skills’ to a balance in the three components.

Feel free to copy the worksheet of the two diagrams and pass out to participants, or draw the model large enough for all participants to see.
Moving Towards a Balanced Mathematics Program

Diagram 1:

Traditional Math Instruction

- Procedural Skills "How the math works"
- Problem Solving "Where the math works"
- Conceptual Understanding "Why the math works"

Diagram 2:

Ideal Math Instruction

- Procedural Skills "How the math works"
- Problem Solving "Where the math works"
- Conceptual Understanding "Why the math works"

1 Adapted from Prentice Hall SB472 Training Manual (August, 2009)
A. Moving towards a balanced math program

Currently, it seems like most mathematics concepts are traditionally taught through the use of procedures (as shown in diagram 1, represented by the larger circle). By mostly teaching procedural skills, the other two components—problem solving and conceptual understanding—become weakened; when students only memorize rules (and are only taught rules), they do not possess the skills to problem solve. In the second diagram, on the other hand, increasing students’ conceptual understanding will in turn increase their problem solving skills. In the second diagram, procedural skills do not have to be represented by a bigger circle because increasing conceptual understanding in the classroom should, in turn, balance the 3 components.

Incorporating conceptual teaching into your classrooms is not intended to diminish procedural skills; in fact, procedural skills are necessary for students to be successful. However, procedural skills should not overpower mathematics teaching. Teaching conceptually should be just as important as teaching procedurally, so mathematics teaching can become balanced. For example, student needs to know the correct procedure for finding the slope between two points, in order to do so. But by increasing their conceptual understanding on the topic, students will understanding the meaning of slope and the procedure will then make sense and be easy to remember.

B. Procedural (computational) skills

Procedural skills refer to the ‘rules’ for solving problems; it involves solving problems routinely and automatically, and practice is required for proficiency. For example, a common math ‘rule’ is: a negative number times another negative number
equals a positive number (i.e. \((-\)(-) = +\)). Teaching the rule only (without any background knowledge or connection to other concepts) could be temporarily beneficial for students, especially if they are asked to complete a worksheet that has them solely multiplying negative numbers together. However, when they learn the procedure only, they are more likely to forget the rule, use it incorrectly, and be unaware of when it is required, than if you show them why the rule is true and where it works. Similarly, when students memorize rules like, “negative times negative equals positive,” they think that whenever they see 2 negative signs together they become positive; however, adding two negative numbers together does not follow the same rule. Students need to connect the rules and procedures of mathematics to something other than pure memorization to be successful.

Part of the reason why the ‘procedural skills’ circle is so much bigger than the other two in diagram 1 is because it seems to be able to stand independently from the other two components. Students can know the procedure without understanding the concept and still get the correct answer (which is probably why this method of teaching is so popular), but there are inherent problems with only teaching procedures and not providing students with the proper problem solving skills and conceptual understanding that is required to be proficient in mathematics.

C. Problem solving

Problem solving refers to ‘where’ the math works—knowing where to use math procedures is just as important as knowing how to use the procedures. In math, problem solving is often found through the use of word problems; in word problems, students
must form words into mathematical representation, and then are required to know which
procedures to use. Problem solving integrates all three components of a balanced math
program—students need to have a conceptual understanding of the procedures needed
and they are required to perform the procedures correctly in order to acquire the correct
answer. For example, consider the problem: “John has $12.50 and owes Sheila $4.75. How
much money will John have after he pays Sheila?” In order to properly facilitate
the mathematical representation of the words in this problem, students need to understand
that ‘owe’—in this example—means ‘subtract.’ Then, they need to know how to
properly subtract decimals in order to acquire the correct answer. Without conceptual
understanding, students will have trouble recognizing that it is a subtraction problem and
will not be able to represent the words mathematically—conceptual understanding seems
to be the link connecting problem solving to procedural skills.

D. Conceptual understanding

Lastly, the third and most important component to creating a balanced math
program is conceptual understanding—in other words, the underlying logic and reasoning
behind “why” mathematics concepts work. Rather than having students simply
memorize rules, it is important that they understand why those rules are true. Most
mathematics concepts are tied to procedures and rules, so it may become easy to only
teach the rules (especially when they produce correct results), but, as discussed earlier,
when students only know the rules and don’t understand the underlying logic behind
concepts, they are more likely to forget the correct procedures, use them incorrectly, and
fail to notice procedural errors.
Teaching conceptually is important for the following 4 reasons, which I will explain in further detail: 1) it promotes deductive reasoning, 2) it facilitates the representation phase of word problems, 3) it has students ‘explain’ concepts which draw upon their higher-order thinking skills, rather than memorizing concepts, which draws on students’ lower-level thinking skills, and 4) it helps students understand given procedures.

First, conceptual understanding promotes deductive reasoning; if students forget the proper procedure, they can figure out how to solve the problem based on their understanding of the concept. For example, a student may forget the rule for absolute value (i.e. \(|-x| = x\)), but if they remember that absolute value represents the distance between a number and zero, and distance is always positive, then they will know that their absolute value answer will also be positive. Similarly, having a strong conceptual understanding behind procedures will allow students to recognize when they make a mistake. As in the example above, students may not realize that getting a negative answer to an absolute value problem must be wrong because they never understood why absolute value always yields positive results.

Second, conceptual understanding helps facilitate the representation phase of word problems. As in an earlier example, students would need to understand that ‘owing’ someone money is the same as ‘subtracting’ mathematically—without understanding why ‘owe’ means ‘subtract,’ students may be likely to forget that subtraction is required. In order for students to use procedures correctly in word
problems, they first need to know why certain procedures are required for certain problems.

Third, conceptual understanding seems to draw upon students’ higher-order thinking skills. While Bloom’s Taxonomy may not often be used to describe mathematics problems and questions, it is still important to consider the kinds of using questions that draw upon students’ higher-order thinking skills. In math, problems that draw upon recall and rote memorization require lower-order thinking—these kinds of problems can often be found in ‘procedural skills’ problems in which students are asked to follow specific rules only. On the other hand, problems that draw on students’ conceptual understanding (i.e. questions that ask them to explain why certain procedures work for certain problems) require higher-order thinking. Math classes do not have to be places where information is simply ‘dumped’ on students—rather, mathematics provides perfect opportunities for creating critical thinkers.

Lastly, conceptual understanding can help students understand procedures. When students truly understand the procedures, they will be more likely to notice when they make an error and will be able to eliminate possible responses to given math problems. For example, on most multiple-choice questions (when four responses are given), students should usually be able to eliminate at least one of the answers solely based on their conceptual understanding of the procedure needed to solve it. The problem below is a good example of conceptual understanding because it has students identify where a mistake was made, and explain why it is wrong:

\[(3 + 2)^2 = 3^2 + 2^2\]
a) What is wrong with this equation?

b) Prove that the equation is *incorrect*

After lunch, you will discuss how teachers can promote conceptual understanding in mathematics classes.

VI. Discussion: Increasing students’ conceptual understanding (≈ 30 mins)

Knowing that conceptual understanding is important to student success in mathematics, participants will now create a list of some possible ways to promote conceptual understanding. There are most likely teachers who are already doing things in their classes to promote conceptual understanding (even if they don’t realize it)—this ‘sharing’ time can be extremely beneficial for participants to acquire realistic teaching strategies that they can use in the future.

The presenter can decide if the discussion will be conducted as a large group or in smaller groups (3 – 4 teachers). If you choose to conduct a whole-group discussion, write responses on a large poster or whiteboard for everyone to see. If participants have their discussions in smaller groups, have a representative from each group share their responses and record them for everyone to see.

Below, I have incorporated a list of techniques to promote conceptual understanding. Welcome any and all suggestions from participants, and after the discussion you can compare the list you have created with the suggestions I offer below. If there are items on the list below that are not mentioned during your discussion, please
Discussion Question:

- Now that we know why conceptual understanding is important to student success in mathematics, how can it be achieved in the classroom?

Collaboratively, create a list of possible ways to increase students’ conceptual understanding in mathematics. Have participants collaboratively create a list first, then you may add any or all of the items below:

1) Use hands-on activities to introduce new topics
2) Use hands-on activities to practice already-introduced topics
3) Connect concepts to students’ backgrounds
4) Connect concepts to real-life applications
   a. For example: problems involving money and sales tax
5) Connect new concepts to past concepts and previous knowledge
6) Use small-group projects to promote concept discovery
7) Use small-group activities to promote discussion (or even conflict) surrounding math concepts
8) Use technology to draw on students’ interests
9) Use everyday materials as manipulatives
   a. For example: decks of cards, dice, marbles, egg cartons, measuring cups, and candy
10) Explain concepts in a way that provides the underlying logic behind the procedures (as opposed to simply stating the rules or formulas)

11) When using worksheets, provide a variety of questions that covers each of the 3 components of a balanced math program

12) Have students create an activity that teaches a given concept

13) Ask questions that draw upon students’ higher order thinking skills, in which they analyze, evaluate, or create information

14) Move away from an overabundance of computational problems for a given topic or unit

15) Have students create their own math problems or entire math worksheets

At this point, please remind participants that conceptual understanding does not ONLY mean incorporating hands-on activities and projects. Procedural skills are still necessary for many mathematics concepts – but it is important to not ONLY teach the computational skills involved with solving math problems. The list above provides some techniques that teachers can use in conjunction with the procedural skills they teach.

BREAK (Lunch – 1 hour)

VI. Activity: Analyzing a math worksheet (≈ 1 hour 30 mins)

For this activity, have participants get into groups of 3 or 4. Each group will be analyzing the same math worksheet to determine how ‘balanced’ it is based on the three components of a balanced math program (procedural skills, problem solving,
and conceptual understanding). The worksheet can be found on page 25, and should be printed out for participants (at least one worksheet per group).

Directions for presenter: “Analyzing a Math Worksheet”

1) You will need a large version of the Venn-diagram representing a balanced math program (see diagram 2 – page 14). You can make it into a poster or draw it largely on the whiteboard.

2) Give each group 6 different-colored post-its, numbered 1 through 6.
   a. The same color for each group will represent the same number (1 – 6)

3) Give each group the worksheet containing 6 math problems (page 25).

4) For each of the 6 problems on the worksheet, group members will decide where each of them fits in the Venn-diagram (i.e. does the problem rely on students’ procedural skills, problem solving skills, conceptual understanding, or a combination of 2 or 3 of them?) Group members should collaborate on each problem and should attempt to reach a consensus for each one.

5) Once they have finished analyzing each problem, one member from each group will come up to the poster to place their post-its on the Venn diagram.

6) There should be obvious clusters of the same color on different sections of the Venn diagram (showing that groups agree on where to place problems). There will most likely be some disagreement for certain problems, but this provides an opportunity for discussion.

Once all the post-its are on the Venn diagram, discuss the nature of the math worksheet; some possible discussion questions are:
Discussion Questions:

- What trends do you see in the Venn diagram?
  - What does this tell us about this worksheet?

- Is this math worksheet balanced?
  - Is there an appropriate amount of procedural skills, problem solving, and conceptual understanding questions?
  - What changes, if any, would you make to this worksheet?
    - What would you add? What would you take away?

- How can you check that your own worksheets and activities are balanced?
Worksheet: Balanced Math Problems

Analyze each math problem to classify it as requiring either problem solving skills, procedural skills, conceptual understanding, or any combination thereof. Use the Venn diagram model to record your ideas. These problems come from Algebra I standards.

<table>
<thead>
<tr>
<th>1) James has made the following mistake. Explain his error and how he can correct it: $3^5 \cdot 3^4 = 9^9$</th>
<th>2) Find 32% of 225</th>
<th>3) Add $3x + 2$ and $3 - 4x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4) One number is 4 times another number. Their sum is 60. What are the two numbers?</td>
<td>5) Solve: $3(5x + 12) = 48$</td>
<td>6) A teacher has 54 research papers to read. She can read three of them in 40 minutes. How long will it take her to read all of the papers at the same rate?</td>
</tr>
</tbody>
</table>
Summary (≈ 15 mins)

This section can simply be a recap of today’s session. It may be helpful to
refer back to the agenda to show that you did, in fact, cover everything you intended. At
this point, you may also want to refer back to their original list of ‘hopes’ for this
training, and address any items on the list that were covered in today’s session and those
that will be covered in the next session. Overall, summarize what you did today and
emphasize any helpful techniques or teaching strategies that participants can leave with.

Today we explored the definition of conceptual understanding and how it fits in to
a balanced math program. We also discussed the importance of conceptual
understanding in achieving a balanced math program, and reviewed some general
strategies that teachers can use in their classrooms. Thank you for contributing ideas for
strategies that you are already using in your classrooms—I hope that you continue to use
techniques that promote conceptual understanding and can incorporate some more into
your teaching.

Hopefully it was helpful to collaborate as a group to analyze the math worksheet
to see how ‘balanced’ it was—this is a technique that you should start doing with your
own worksheets and activities to make sure that the problems you use span different parts
of the Venn-diagram. If you are not giving students problems that draw upon their
problem solving skills and conceptual understanding, they will not think that they ever
need to understand mathematics beyond memorizing procedures.
VII. Closure (≈ 15 mins)

I hope that you found today’s session valuable and can start thinking about how you can incorporate conceptual understanding into your lessons. Next time you will get in groups to create lessons that contain activities for promoting conceptual understanding on certain math topics that our students struggle with.

Homework: For next time, I would like you to think about the two questions below:

1) What specific concepts that you teach are *most* important to student success? (i.e. if you could only teach your students a few concepts for the entire year, what would you choose that would allow them to be successful in subsequent math classes?) Create a list of 4 – 5 items.

2) What concepts do your students struggle with the *most*? Create a list of 4 – 5 items. (Some responses may be the same for questions 1 and 2)

Be prepared to share your responses at the start of next session.
Day 2: Agenda

I. Recap of Day 1………………………………………………………… 8:30 – 9:00

II. Goals for today ……………………………………………………………9:00 – 9:05

III. Mathematics concepts: what’s important? …………………………..9:05 – 10:00

IV. Activity: creating an introductory lesson ……………………………10:00 – 12:00

LUNCH

V. Share activities……………………………………………………………1:00 – 2:30

VI. Summary of today’s session…………………………………………2:30 – 2:45

VII. Closure/Homework……………………………………………………2:45 – 3:00

Main objectives for participants:

1) To recall the importance of conceptual understanding

2) To discuss the most important concepts in secondary mathematics courses

3) To discuss the concepts that students tend to struggle with the most

4) To create, in a small group, an introductory lesson plan that promotes conceptual
   for one of the ‘most important’ concepts

5) To demonstrate lessons to the rest of the group
I. Recap of Day 1 (≈ 30 mins)

Start today’s session with a recap of the last session. This will be especially important if the sessions are not given on consecutive days, and will help catch up any individuals who missed the first session. Ideally, the same participants will be present at all sessions, but that probably will not be the case.

Use this time to emphasize the main points that emerged from last session:

1. The definition of conceptual understanding and how it fits in a ‘balanced math program’ (see page 17)
2. The importance of conceptual understanding in secondary mathematics (see page 17 – 19)
3. Some ideas about how conceptual understanding can realistically be achieved in the classroom (refer to the list created yesterday)
4. How to analyze math problems to determine where they fall in the Venn-diagram of a balanced math program

You may want to show the Venn-diagram again of a balanced math program.

Stress the importance of how mathematics is traditionally taught (i.e. through procedures) versus how mathematics should be taught to best fit the needs of our students (i.e. through a balance of procedures, conceptual understanding, and problem solving).

Invite discussion and questions regarding the previous session.
II. Goals for today (≈ 5 mins)

Once you have sufficiently recapped last session, state the goals for today’s session.

Today you will:

1. Recall the importance of conceptual understanding in secondary mathematics teaching.
2. Discuss with your peers, the concepts that seem to be most important in your math classes and those that students struggle with.
3. Create (in a group) a lesson plan that promotes conceptual understanding on a certain topic.
4. Demonstrate your lesson to the rest of the class.

III. Mathematics concepts: What’s Important? (class discussion ≈ 55 mins)

At the end of the last session, I asked you to consider which concepts are most important that you teach in your classes (i.e. what students really need to master in order to be successful in mathematics) and which concepts students struggle with. Your responses to the 2 questions may overlap and be similar, which is fine. We will create a list of the math concepts that you came up with:

Record participants’ responses on a big poster paper or whiteboard for everyone to see:

- Possible responses for questions 1 and 2 (or some to add to the list):
  - Fractions
Not all participants may agree on some topics (for example, perhaps Geometry teachers do not think that percentages are an important concept for students to master in their course) – but that’s ok! Concepts that are important to some teachers may not be important to others; the point is that teachers recognize which concepts are most important to their course specifically.

Invite discussion on the list of topics. Teachers may agree that some of the concepts on the list do not belong there. Work collaboratively on the list until there is group consensus on the ‘important concepts’ for mathematics teaching.

Keep the list posted because you will use it for the following activity.

IV. Activity: Creating an introductory lesson (≈ 2 hours)

For this activity, participants will get into groups of 3 or 4 with others who teach the same subject area that they do (for example, Algebra I teachers together, Geometry teachers together, etc.) That way, their lesson will be course specific and they can actually use the lesson that they create here in their own classes.
Provide some materials for participants to use to create their lessons, so they are not restricted to solely using paper and pencil. Also, simply seeing the abundance of materials may spark some creativity among teachers that they may not have considered without having the materials easily and readily available.

Some examples of materials that you can bring are:

- Cards
- Dice
- Empty egg cartons
- Rulers
- Marbles
- Candy
- String
- Index cards
- Markers
- Pie dishes
- Measuring cups
- Colored paper
- Money (fake?)
- Chess pieces
- Rubber bands
- Algebra tiles (if accessible)
- Individual whiteboards
- Calculator

Directions for the activity are given on the following page. You may post the directions for participants, or print the following page and hand out the directions to each group.

Print and pass out the Lesson Plan Template (page 50 – 51) to each group, or use a site-based or district-based lesson plan template, if one exists and you prefer to use it.
Creating an Introductory Hands-On Activity that Promotes Conceptual Understanding

Directions:

1. Get into groups of 3 or 4 with other teachers from your subject area.
   - If you teach more than one subject area, pick one to focus on for today.

2. Pick one of the ‘important concepts’ from the list we created that you would like to focus on for your lesson.
   - Come to a consensus with your group on the concept you would like to focus on (it should relate directly to your subject matter).

3. Though hands-on activities are not the only way to promote conceptual understanding, today you will focus on creating a hands-on activity as a way to promote conceptual understanding.
   - You may incorporate any of the materials provided in your activity.
   - Follow the lesson plan template for this activity
   - You will create a one-day lesson that *introduces* the topic you are focusing on.

4. Include a 1-paragraph rationale for how the hands-on activity promotes students’ conceptual understanding.

5. You can decide if your activity will be geared towards small groups of students or created for a whole-class activity, but make sure that communication and talking are involved. (In other words, it should *not* look like a student completing a worksheet independently). The activity should promote discussion and collaboration.

6. Later, you will share your presentation with the rest of the class.
V. Sharing Activities (≈ 1 hour 30 mins)

Each group will share their lesson with the rest of the class. Here are some basic guidelines for how groups could present their lessons to the class:

- What course is this geared toward?
- What concept did you pick to focus on?
- Go through your lesson step-by-step (use the questions below as a guide. Not all questions may apply):
  - What previous knowledge do students need for this topic?
  - What are the lesson objectives?
  - What is the anticipatory set (warm-up)?
  - What input will you offer students for this lesson?
  - Explain your hands-on activity:
    - Give a detailed explanation of the activity, including the materials used, how students will be grouped, how long the activity should take, and what the teachers role is during the activity.
    - Demonstrate your activity to the other participants, as if you were instructing your students in class.
  - Is there homework that accompanies this lesson?
  - How will you assess students’ understanding on this topic?
Participants will hopefully find their peers’ lessons valuable and be able to use others’ hands-on activities in their classes in the future. Participants should take notes on the other groups’ lessons so they leave this training with a list of hands-on activities to use in their classrooms that promote conceptual understanding. If a copier is accessible, make copies of each groups’ introductory lessons for the other teachers to take with them.

VI. Summary of today’s session (≈ 15 mins)

Today you collaboratively created a list of the ‘most important’ concepts in mathematics teaching—in other words, those concepts that seem to be key for student success and also those concepts that students tend to struggle with. Some concepts that are important in your subject area (for example, geometry) may not necessarily be as important in another subject area (for example, algebra) but there should be an obvious connection between the various concepts that you came up with and also an apparent overlap between courses. Recognizing which concepts are most important to your students specifically will be helpful for deciding when to use hands-on activities, technology, projects, and other strategies for teaching conceptually.

You also created an introductory lesson plan that focused on a topic that you identified as important for student success in your subject area. Your introductory lesson incorporated a “conceptual understanding” component in the form of a hands-on activity. The lesson plan that you created with your peers, among with other groups’ lessons,
should serve as a jumping off point for promoting conceptual understanding in your classes.

VII. Closure (≈ 15 mins)

At this point, revisit the agenda and objectives from Day 2 (page 28). Take this time to answer any unanswered questions that participants had from today’s training session. If you have time, it may be helpful to pose the following questions to participants:

- Do you have any questions, comments, or concerns regarding what you accomplished today?
- Did you accomplish what you set out to do in today’s session?
- Did you meet the outlined objectives for today?
  - If not, why?
  - What could have been done differently in order for you to meet the objectives?

Homework: For tomorrow, please think about the following questions, and come to class ready to share your responses:

- Is there a difference between introductory and follow-up lessons?
- If so, what are the main differences between the two?
- What are the similarities between introductory and follow-up lessons?
Day 3: Agenda

I. Recap of Day 2 ......................................................... 8:30 – 8:45
II. Goals for today ....................................................... 8:45 – 8:50
III. Discussion: introductory vs. follow-up lessons .............. 8:50 – 9:20
IV. Activity: creating a follow-up lesson ........................... 9:20 – 11:00
V. Share activities (start) ................................................ 11:00 – 12:00

LUNCH
Share activities (continue) ............................................. 1:00 – 1:45

VI. Summary of the 3 days of training ............................... 1:45 – 2:00
VII. Looking to the future ................................................ 2:00 – 2:30
VIII. Evaluation ............................................................. 2:30 – 2:45
IX. Closure ................................................................. 2:45 – 3:00

Main objectives for participants:

1) To compare and contrast various elements typical of an introductory lesson and those typical of a follow-up lesson

2) To create a follow-up lesson plan that builds upon the elements presented in yesterday’s introductory lesson plan

3) To demonstrate the follow-up lesson to the rest of the group

4) To predict what techniques, specifically, you will incorporate into your mathematics classes

5) To evaluate the 3 days of training
I. Recap of Day 2 (≈ 15 mins)

Start today’s session with a recap of the last session. Participants will be building off of the lesson plans they created yesterday, so a recap of the main points from yesterday’s session should help get them in the right mind-set and ready to continue working. Use this time to emphasize the main points presented in the previous session:

1. The “important” mathematics concepts (i.e. those that seem to be important for student success and also those that most students tend to struggle with.)
2. You created a hands-on activity as part of an introductory lesson that focused on an important mathematics concept.
3. You shared your lesson plans with the group, and now you have a compilation of lesson plans on various mathematics topics to take with you.
4. Invite discussion surrounding the previous session, and take this time to clarify any misunderstandings that participants may have regarding the lesson plans they created during the previous session.

II. Goals for today (≈ 5 mins)

State the goals for today’s session, referencing the objectives found on the previous page.

Today you will:

1) Compare and contrast elements found in introductory lessons with those found in follow-up lessons.
2) Create a follow-up lesson plan that builds upon the elements presented in yesterday’s introductory lesson plan.

3) Demonstrate your follow-up lesson to the rest of the group.

III. Discussion: Differences between introductory and follow-up lessons (≈ 30 mins)

At the end of the previous session, participants were asked to consider the differences, if any, between introductory lessons and follow-up lessons. Allow participants to openly discuss their thoughts in a whole-class discussion. This discussion topic is meant to get participants thinking about how the activity that they will create today should be different than (or, perhaps, similar to) the one they created yesterday. Use the following questions as a guide to run the discussion. To make the discussion more visual for participants, you can create a table (or a Venn-diagram, as shown below) on a poster board or whiteboard for everyone to see.

For example:

![Venn Diagram](image-url)
Discussion Questions:

1) What kinds of elements are typical of an introductory lesson in mathematics?

2) What kinds of elements are typical of a follow-up lesson (in which the topic has already been introduced) in mathematics?

3) In what ways are introductory lessons similar to follow-up lessons?

4) In what ways are introductory lessons dissimilar to follow-up lessons?

Use the ideas presented from this discussion regarding the similarities and differences of introductory lessons and follow-up lessons to help guide the creation of your follow-up lesson.

IV. Activity: Creating a follow-up lesson (≈ 1 hour 40 mins)

For this activity, participants will get into the same groups they were in during the previous session’s activity, in which they created an introductory lesson. Any participants who missed the previous day’s training should join a group based on his or her subject area. Please provide the same materials that were available to participants for the introductory lesson-plan activity (see list on page 32). On the following page are the directions for the follow-up lesson plan activity. Please post the directions or print out the page to give to participants.

Also, provide the same lesson plan template that was given during the previous session. (The one provided here is found on page 50 – 51).
Creating a Follow-up Hands-On Activity that Promotes Conceptual Understanding

Directions:

1. Get into the same group of 3-4 teachers from the previous session’s activity, in which you created an introductory lesson.

2. Continue with the same mathematics concept that you focused on in your previous lesson.

3. Create a lesson plan that includes a hands-on activity that follows the introductory lesson plan you created previously.

   - The topic of this lesson should be the same as your introductory lesson, but you may explore further into the concept than you had previously.

     - For example, an introductory lesson on adding fractions might solely focus on adding fractions with like denominators, while a follow-up lesson might include adding fractions with unlike denominators.

   - You can decide if this activity should be conducted directly following the introductory activity (for example, the next day), or if there should be other lessons in between the two activities.

4. Use any of the materials provided to assist you in creating your hands-on activity.

   - You do not have to use the same materials you used in the previous activity.

5. Follow the lesson plan template

6. Later, you will share your activity with the rest of the class.
If time permits, groups can begin sharing their activities before the lunch break. You can continue sharing after lunch until all groups have demonstrated their hands-on activity to the group.

BREAK (Lunch – 1 hour)

V. Share activities (≈ 1 hour 45 mins)

Each group will now share their lesson with the rest of the class. Here are some basic guidelines for how groups could present their lessons to the class:

- What course is this geared toward?
- What concept are you focusing on?
- Give a short recap of your introductory hands-on lesson
- Explain why this activity is best used as a follow-up lesson, and not as an introductory lesson.
- Will there be any lag time in between activities (days, weeks, etc.) or should this activity be conducted immediately following the introductory activity?
- Go through each step of your lesson plan template (use the questions below as a guide. Not all questions may apply):
  - What previous knowledge do students need for this topic?
  - What are the lesson objectives?
  - What is the anticipatory set (warm-up)?
  - What input will you offer students for this lesson?
  - Explain your hands-on activity:
- Give a detailed explanation of the activity, including the materials used, how students will be grouped, how long the activity should take, and what the teacher’s role is during the activity.

- Demonstrate your activity to the other participants, as if you were instructing your students in class.
  
  Is there homework that accompanies this lesson?
  
  How will you assess students’ understanding on this topic?

As during the previous session, if a copy machine is available to you, please make copies of the groups’ lesson plans so all participants can leave with various lessons on different topics.

VI. Summary of the 3 days of training (~ 15 mins)

After all groups have shared and demonstrated their hands-activities to the rest of the class, and teachers now have a collection of lesson plans that promote conceptual understanding on various topics in mathematics, you can summarize what you have accomplished as a group over the past 3 sessions. Refer to each day’s objectives, and show participants how they accomplished all of the goals that this training set out for them to do.

On the first day of training you:

- Listened to a lecture on the importance of conceptual understanding, in order to understand why it is necessary for many students to succeed in mathematics.
Identified pedagogy representative of “traditional” secondary mathematics teaching, in order to realize that most traditional math classes are missing the conceptual understanding component.

Analyzed a mathematics worksheet to see how ‘balanced’ it was, so you can identify math problems as computational, conceptual, or problem-solving.

On the second day of training you:

- Identified concepts in mathematics that seem to be most important for success in a given class, and also concepts that students typically struggle with.
- Collaborated with other teachers who teach the same subject as you, and created a hands-on activity for an introductory lesson that focused on one of the “important concepts”.
- Demonstrated your activity to the rest of the group, and acquired a collection of other groups’ introductory lesson plans.

On the third day of training you:

- Compared and contrasted elements of introductory lessons and follow-up lessons, as a way to determine how your follow-up activity may be similar or dissimilar to your introductory activity.
- Created a follow-up hands-on activity on the same topic as your introductory lesson, with the same group of teachers you worked with previously.
- Demonstrated your activity to the rest of the group, and acquired a collection of other groups’ follow-up lesson plans.
Overall, the main objectives of these training sessions set out to have you understand the importance of conceptual understanding, reflect on and analyze your own teaching and “traditional mathematics” teaching, create lesson plans that promote conceptual understanding, and predict how you will realistically incorporate conceptual understanding into your classes. Before you leave here today, you will consider how you plan to incorporate conceptual understanding in your own classes.

VII. Looking to the future (≈ 30 mins)

*If this training has been effective, teachers will begin to incorporate techniques into their classes that promote conceptual understanding. They will hopefully move away from solely teaching procedural skills, to incorporating conceptual understanding and problem solving techniques. In their classrooms, they will recreate the lessons that they created here—and, hopefully, other lessons from the various groups. Take this time to allow participants to share with the group what they specifically plan to do differently in their classrooms because of this training.*

*This can be conducted easily as a whole-class discussion by posing the following questions to the group, and allowing them to take turns answering. *Feel free to alter or omit questions that may not apply to participants.*

Discussion Questions:

- What specific techniques or activities from these training sessions do you plan to incorporate into your own teaching?
- What, from these training sessions, was most useful to you?
What do you plan to change, on a day-to-day basis about your current pedagogy in order to teach more conceptually?

VIII. Evaluation (≈ 15 mins)

Once the discussion has died down, take a few minutes to have participants fill out the evaluation form (found on pages 52-53). The evaluation form is meant to provide constructive feedback on the three days of training. Trainers can use the information from the evaluation forms to shape how subsequent professional development trainings should be conducted.

IX. Closure (≈ 15 mins)

Take this time to wrap-up the 3 days of training. Refer back to the “Macro Objectives for Professional Development on Conceptual Understanding” (page 6), so participants can realize how the past few days of training can affect how they will teach mathematics in the future. Invite discussion surrounding the various macro objectives (see Sample Questions below).

Sample questions to invite discussion surrounding macro objectives:

- Do you buy into the idea the conceptual understanding in mathematics is critical for student success?
- What will you change, if anything, about the way you teach mathematics?
- How do you think the overall culture in your classroom will change since these trainings?
What long-term effects will this training have on the way you teach mathematics?

Below is a “conclusion” for the 3 days of training. Please read through ahead of time and use the main points presented here to end the training on a positive note. Feel free to modify the conclusion below, especially if you had a specific focus for this training (for example, on English language learners). You can also use any remaining time to revisit any of the discussions or activities that participants needed more time on.

Hopefully you will actively incorporate some of the ideas presented over the course of this professional development training in your classes. Remember, if we are not teaching conceptually (i.e. through the use of hands-on activities, real-life applications, technology, and small group collaboration), then students are most likely not understanding conceptually.

Over the next few months, try and incorporate at least one creative activity per week that promotes students’ conceptual understanding. It does not necessarily have to be a hands-on activity—for example, consider using small groups in the classroom more often, try to relate math concepts to students’ interests, or possibly take the students to the computer lab for a class period. Invite discussion in your classroom surrounding explanations of mathematical concepts and procedures, rather than simply showing or telling students a set of rules to follow to acquire the correct answer.
If you focus on the ‘most important concepts’ and teach those concepts conceptually as opposed to procedurally, I think you will start to see an improvement in student success in mathematics.
Appendix

Lesson plan template.................................................................50
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Lesson Plan Template

What background knowledge do your students have on this topic?

---

q Objectives:

q Anticipatory set (warm-up):

q Input/Modeling (what do students need to know for this activity?):

q Hands-on activity:
  - What materials do you need?
  - Will the students be split up in groups?
● Explain the activity in full detail below, including directions for students:

q Independent practice (homework?)

q Closure

-----------------------------------------------------------------------------------------------------------

● How will you check for understanding during the lesson/activity?

● How will you monitor your students’ progress?

● How will you assess your students on this topic?
Evaluation Form

Event: ________________________________       Date: _________________

For each item below, provide a 1 to 5 rating by circling a number after each statement.
1= strongly disagree   2= disagree   3= neutral   4= agree   5= strongly agree

Overall Program:
1. The total program was of high quality. 
   1  2  3  4  5

2. The program content will be useful to me. 
   1  2  3  4  5

Comments: ____________________________________________
_______________________________________________________

Impact:
3. I will use knowledge and skills gained during this professional development event to impact student learning. 
   1  2  3  4  5

4. I would like additional opportunities to expand my new knowledge and skills. 
   1  2  3  4  5

Comments: ____________________________________________
_______________________________________________________

Professional Development Practices:
5. A supportive climate of professional community was created. 
   1  2  3  4  5

6. Opportunities to network and learn from colleagues were supported. 
   1  2  3  4  5

7. The opportunity to seek meaning and construct new knowledge was provided. 
   1  2  3  4  5

8. An appropriate balance between presentation and interaction was achieved. 
   1  2  3  4  5
Comments:______________________________________________________________
________________________________________________________________________
________________________________________________________________________
Presenter(s):
9. The presenter(s)’s overall effectiveness was high. 1 2 3 4 5
10. The content of the presenter’s presentation was useful. 1 2 3 4 5
11. The presenter(s) used appropriate instructional techniques. 1 2 3 4 5
12. The presenter(s) used high-quality materials. 1 2 3 4 5
Comments:______________________________________________________________
________________________________________________________________________
Feedback Questions:
What did you value most from this professional development event?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
What will you use from this professional development event in your own classroom setting?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
What other professional development events would be of benefit to you in the future?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
*Contact Information (optional):
Name:  _________________________________________________________________
E-mail:  ________________________________________________________________
Bloom’s Taxonomy and Writing Objectives

Bloom’s “taxonomy of educational objectives” is a framework for classifying statements of “what we expect or intend students to learn as a result of instruction” (Krathwohl, 2002). Instructors often use Bloom’s taxonomy as a hierarchy, in order to identify “lower-level thinking” (as outlined in the beginning two tiers of the framework, in which students remember and understand information) and “higher-level thinking” (as outlined in the upper two tiers of the framework, in which students evaluate and create information). The following chart on Bloom’s Taxonomy is a helpful tool for writing objectives. Teachers should consider the verbs they use when writing objectives (as outlined under “Key Words” in the right-hand column) in order to properly identify the level of thinking they are requiring from their students.

Activities that promote conceptual understanding often lie within the upper two-thirds of Bloom’s taxonomy, in which students apply, analyze, evaluate, and create. When creating lesson plans and writing objectives, teachers must consider the words they use and the level of thinking they are promoting. Activities that promote conceptual understanding in mathematics should, therefore, require students to apply a concept in a new situation, analyze material as a way to understand its structure, evaluate the value of certain procedures, or create something that builds upon a concept’s diverse elements.

The objectives written for this professional development training move upward through Bloom’s taxonomy—at the beginning of the training, participants are asked to remember and understand the importance of conceptual understanding, then they move on to analyzing a mathematics worksheet, and finally participants complete the training
by creating lesson plans that promote conceptual understanding.

<table>
<thead>
<tr>
<th>Level of Thinking*</th>
<th>Key Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember: Recall data or information</td>
<td>The verbs below describe students’ mental actions</td>
</tr>
<tr>
<td>Understand: Understand the meaning, translation, interpolation, and interpretation of instructions and problems. State a problem in one’s own words.</td>
<td>Key Words: define, describe, identify, know, label, list, match, name, outline, recall, recognize, reproduce, select, state</td>
</tr>
<tr>
<td>Apply: Use a concept in a new situation or unprompted use of an abstraction. Applies what was learned in the classroom into novel situations in the work place.</td>
<td>Key Words: comprehend, convert, defend, distinguish, estimate, explain, extend, generalize, give examples, infer, interpret, paraphrase, predict, rewrite, summarize, translate.</td>
</tr>
<tr>
<td>Analyze: Separates material or concepts into component parts so that its organizational structure may be understood. Distinguishes between facts and inferences.</td>
<td>Key Words: analyze, break down, compare, contrast, diagram, deconstruct, differentiate, discriminate, distinguish, identify illustrate, infer, outline, relate, select, separate</td>
</tr>
<tr>
<td>Evaluate: Make judgments about the value of idea of materials.</td>
<td>Key Words: appraise, compare, conclude, contrast, criticize, critique, defend, describe, discriminate, evaluate, explain, interpret, justify, relate, summarize, support</td>
</tr>
<tr>
<td>Create: Builds a structure or pattern from diverse elements. Put parts together to form a whole, with emphasis on creating a new meaning or structure.</td>
<td>Key Words: categorize, combine, compile, compose, create, devise, design, explain, generate, modify, organize, plan, rearrange, reconstruct, relate, reorganize, revise, rewrite, summarize, tell, write.</td>
</tr>
</tbody>
</table>

*Revised categories from:

For more detailed information on conceptual understanding in mathematics, or for questions or comments regarding this handbook, please contact:

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REFERENCES


